Experimental Quantum Cloning of Single Photons

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Although perfect copying of unknown quantum systems is forbidden by the laws of quantum mechanics, approximate cloning is possible. A natural way of realizing quantum cloning of photons is by stimulated emission. In this context, the fundamental quantum limit to the quality of the clones is imposed by the unavoidable presence of spontaneous emission. In our experiment, a single input photon stimulates the emission of additional photons from a source on the basis of parametric down-conversion. This leads to the production of quantum clones with near-optimal fidelity. We also demonstrate universality of the copying procedure by showing that the same fidelity is achieved for arbitrary input states.

No device is capable of producing perfect copies of an unknown quantum system. This statement, known as the "no-cloning theorem" (*1*, *2*), is a direct consequence of the linearity of quantum mechanics and constitutes one of the most significant differences between classical and quantum information. The impossibility of copying quantum information without errors is at the heart of the security of quantum cryptography (*3*). If one could perfectly copy arbitrary quantum states, this would make it possible to exactly determine the state of an individual quantum system, which—in combination with quantum entanglement—would even lead to superluminal communication (*4*). Thus, the no-cloning principle also ensures the peaceful coexistence of quantum mechanics and special relativity.

Given that perfect cloning is impossible, it is natural to ask how well one can clone. This question was first addressed by Bužek and Hillery in (*5*) and initiated a large amount of theoretical work. In particular, bounds on the maximum possible fidelity of the clones produced by universal cloning machines were derived (*6*). A universal cloning machine produces copies of equal quality for all possible input states. After the work of Bužek and Hillery (*5*), quantum cloning was discussed mainly in the language of quantum computing, where its realization was envisioned in the form of a certain quantum logical network consisting of a sequence of elementary quantum gates. An implementation of the cloning network based on nuclear magnetic resonance (NMR) has recently been reported (*7*), but that work by Cummins *et al.* uses ensemble techniques and thus does not constitute true cloning of individual quantum systems. In another experiment, the polarization degree of freedom of a single photon was approximately copied onto an external degree of freedom of the same photon (*8*). Although formally this is a realization of a quantum cloning network, only a single particle is involved in the whole process.

One might look for more natural ways of realizing quantum cloning. In the first papers on the topic, a connection to the process of stimulated emission was made and it was suggested that stimulated emission might allow perfect copying (*4*). It was subsequently pointed out (*9*, *10*) that perfect cloning is prevented by spontaneous emission. Recently, it was proposed (*11*) that optimal quantum cloning, where the quality of the copies saturates the fundamental quantum bounds, could be realized for photons with the use of stimulated emission in parametric down-conversion. First indications of the effect were reported in (*12*), but neither universality nor optimality were demonstrated. We present a demonstration of universal cloning for individual quantum systems, realizing the proposal of Simon, Weihs, and Zeilinger (*11*) and achieving a quality of the clones that is close to optimal.

Universal cloning by stimulated emission proceeds by sending a single input photon into an amplifying medium capable of spontaneously emitting photons of any polarization with equal probability. This rotational invariance of the medium ensures the universality of the cloning procedure (*11*). As a result of stimulated emission, the medium is more likely to emit an additional photon of the same polarization as the input photon than to spontaneously emit a photon of the orthogonal polarization. The probabilities for stimulated and spontaneous emission are always proportional, making it impossible to suppress spontaneous emission without also affecting the stimulated process. Thus, it is spontaneous emission that limits the achievable quality of the quantum cloning and ensures that the no-cloning theorem is not violated (*9*–*11*).

In our experiment (Fig. 1), a strong pump light pulse propagates through a nonlinear crystal, where, with low probability, photons from the pump pulse can split into two photons of lower frequency (a process known as parametric down-conversion). Under suitable conditions and for certain specific directions of emission, the two created photons are entangled in polarization (*13*). The situation can be described by a simplified interaction Hamiltonian

$$H = \kappa(a^\dagger_{\rm v} b^\dagger_{\rm h} - a^\dagger_{\rm h} b^\dagger_{\rm v}) + h.c. \qquad (1)$$

where κ is a coupling constant and $a^\dagger$ and $b^\dagger$ are creation operators for photons in the spatial modes corresponding to two different directions of emission (Fig. 1). The subscripts v and h refer to vertical and horizontal polarization, and *h.c.* is the hermitian conjugate. The Hamiltonian can be shown to be invariant under joint identical polarization transformations in modes *a* and *b*, ensuring that the cloning will be equally good in every polarization basis.

The input photon arrives in mode *a* passing through the nonlinear crystal. Because of the rotational invariance of the Hamiltonian, it is sufficient to consider one particular initial polarization state, for example, $a^\dagger_{\rm v}|0\rangle = |1,0\rangle_a$, where we have introduced the notation $|k,l\rangle_a$ for a state containing *k* vertically and *l* horizontally polarized photons in mode *a*. Its time evolution is obtained by applying the operator e^{-iHt}. For small values of κt, corresponding to the experimental situation, this can be expanded into a Taylor series. The zeroth order term corresponds to the case where no additional photons are produced. This emphasizes that our cloning machine has a probabilistic aspect; sometimes it will just output the input photon. The first order term leads to the following (unnormalized) three-photon state

$$- i\kappa t\,(a^\dagger_{\rm v} b^\dagger_{\rm h} - a^\dagger_{\rm h} b^\dagger_{\rm v})a^\dagger_{\rm v}|0\rangle = - i\kappa t\,(2|2,0\rangle a|0,1\rangle b - |1,1\rangle_a|1,0\rangle_b). \qquad (2)$$

Recall that $|2,0\rangle_a\,|0,1\rangle_b$ is the (normalized) state with two photons in mode $a_{\rm v}$ and one photon in mode $b_{\rm h}$, whereas $|1,1\rangle_a\,|1,0\rangle_b$ has one photon each in modes $a_{\rm v}$, $a_{\rm h}$, and $b_{\rm v}$. The factor $\sqrt{2}$ shows that the additional emitted photon in mode *a* is more likely by a factor of two to be of the same polarization as the input photon than of the orthogonal polarization. In this way, the information about the input

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photon polarization is imprinted on the down-converted photon.

The two photons in mode a are the clones. In the present ideal case, the input photon and the additional photon created in the process have identical space-time wave functions and are thus completely indistinguishable from each other. Therefore, the two photons are both approximate copies of the input photon with the same fidelity. Operationally, the fidelity is defined by picking one of the two photons in mode a and determining with which probability its polarization is identical to that of the input photon. Inspection of the output state in Eq. 2 shows that with a probability of 2/3, both photons are vertically polarized, i.e., they are perfect clones, whereas with a probability of 1/3, the photons have opposite polarization. Therefore, in this case the probability of picking a vertical photon is just 1/2, and the overall fidelity of the clones is given by

$$F = \frac{2}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} = \frac{5}{6} \qquad (3)$$

which has been shown to be the optimal achievable fidelity for the universal cloning of a single photon (*6*). Because of the rotational invariance of the Hamiltonian in Eq. 1, every other input polarization is copied with the same fidelity.

The stimulation effect occurs only when there is overlap between the incoming photon and the photon produced by the source. In our experiment, we use photons created in short pulses. By changing the relative delay between the input photon and the photon created in the down-conversion process, we can continuously vary the degree of distinguishability. Suppose the state of the incoming photon, $\tilde{a}^{\dagger}_{v}|0\rangle$, does not overlap with the down-conversion mode a. The same calculation as above would then lead to a three-photon state

$$-\,i\kappa t\,(a^{\dagger}_{v}b^{\dagger}_{h} - a^{\dagger}_{h}b^{\dagger}_{v})\tilde{a}^{\dagger}_{v}|0\rangle = -\,i\kappa t\,(|1,0\rangle_{a}|1,0\rangle_{\tilde{a}}|0,1\rangle_{b} - |0,1\rangle_{a}|1,0\rangle_{\tilde{a}}|1,0\rangle_{b}) \qquad (4)$$

If $\tilde{a}$ differs from a only by a time delay that is small compared with the time resolution of the detectors (which is of the order of 1 ns), then they are practically though not fundamentally indistinguishable. In this case, the state in Eq. 4 will be experimentally indistinguishable from the state $-i\kappa t\,(|2,0\rangle_{a}\;|0,1\rangle_{b} - |1,1\rangle_{a}|1,0\rangle_{b})$. There is an important distinction with respect to Eq. 2; namely, the factor $\sqrt{2}$ in the first term has disappeared, which means that the additional emitted photon is now equally likely to be vertically or horizontally polarized. There is no stimulation effect.

So far, the third photon that is produced into mode b has played no role in our discussion. However, it serves an important purpose in the experiment as a trigger. As the down-conversion photons are created in pairs, the detection of the photon in mode b means that a clone has indeed been produced in mode a. For our experimental setup, the mere detection of two photons in mode a does not ensure that cloning has indeed occurred because both photons could have been contained in the input pulse. Because the input pulse has an average photon number of only 0.05 and the down-conversion process occurs only with a probability of the order of 1/1000, total photon numbers larger than three are exceedingly unlikely. The possible presence of more than one photon in the input pulse leads to a slight overestimation of the cloning fidelity (by about 0.003). However, this effect is negligible compared with the experimental and statistical errors. As a consequence of the anticorrelation in polarization between the photons in modes a and b, the photon in mode b is actually an optimal anticlone of the input photon (*11*, *14*). Even if the phase between the two terms in the Hamiltonian Eq. 1 is not fixed such that the entanglement between modes a and b is reduced, the cloning procedure will still be universal and work with optimal fidelity, as long as the source emits photons of any polarization with equal probability. However, the quality of the anticlones will steadily decrease as the quantum correlations are lost.

In the experiment, the polarization of the photons in spatial mode a is analyzed, triggered by the detection of a photon in mode b, while varying the overlap between the input photon and the photon created in the crystal. The polarization analysis is performed as follows: For linear polarizations, a λ/2 wave plate is used to select the measuring basis A polarizing beam splitter (PBS) is used to measure the events in which the two photons in mode a have different polarizations [$N(1,1)$], while a polarizer followed by an ordinary beam splitter (BS) is used to probabilistically detect the presence of two identical photons in mode a [$N(2,0)$]. In the case of circular polarization, a λ/4 plate is used to convert circular to linear polarization and subsequently the method for linear polarizations is used. In practice, the PBS is effectively changed into a BS by introducing an additional λ/4 to introduce minimum changes to the experimental setup.

According to our discussion here and by comparing Eqs. 2 and 4, an enhancement of the rate $N(2,0)$ of events where both photons have the same polarization is expected as soon as the input photon and the produced photon overlap. In contrast, there should be no enhancement for the rate $N(1,1)$ of detections where the two photons have orthogonal polarizations because the amplitude is always $i\kappa t$. Moreover, the stimulation effect should be equally strong for all incoming polarizations.

These expectations are fulfilled in the experiment. Figure 2 shows our experimental quantum cloning results. One sees an increase in the $N(2,0)$ count rate in the overlap region. This increase is observed for three complementary input polarizations (linear 0°, linear 45°, and circular left-handed), thus demonstrating universal-

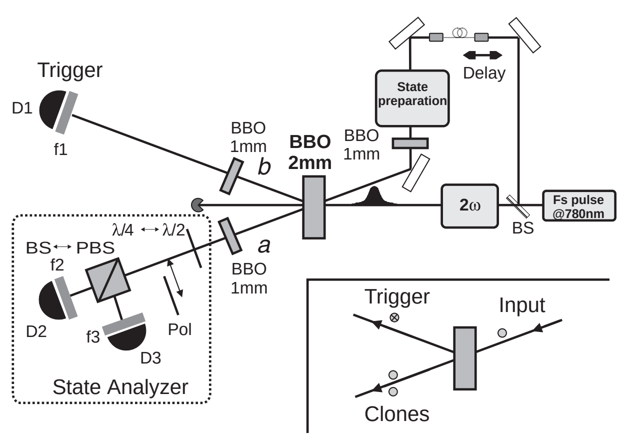

Fig. 1. Setup for cloning by stimulated emission. A Ti:sapphire laser produces light pulses of 120-fs duration, centered at a wavelength of 780 nm. A tiny part of each pulse is split off at the beam splitter BS and then attenuated below the single-photon level, thus probabilistically preparing the input photon. Its polarization state can be adjusted at will. The major part of every pulse from the laser is frequency-doubled and used to pump the nonlinear crystal [β-barium-borate (BBO), 2 mm], where photon pairs entangled in polarization are created into the modes a and b. A delay line containing a single-mode optical fiber facilitates superimposing the input photon and the a photon produced in the crystal. For perfect overlap, the two photons in mode a after the crystal are indistinguishable and both are optimal clones of the input photon. Their polarization is analyzed with the use of wave plates, a polarizer, and a PBS in front of detectors D2 and D3. The photon in mode b serves as a trigger, indicating that parametric down-conversion has occurred. The interference filters f1, f2, and f3 help to increase the overlap between input and down-conversion photons. The three auxiliary crystals (BBO, 1 mm) compensate for birefringence in the nonlinear crystal. The inset illustrates the cloning process. Both clones are in the same mode.

ity. Far away from the overlap region, the probabilities $p(2,0)$ and $p(1,1)$ are actually the same due to the rotational invariance of the source, which has been verified independently. The measured values for the $N(2,0)$ and $N(1,1)$ base levels in Fig. 2 are different because the two identically polarized photons in the $N(2,0)$ case can be detected probabilistically only by observing coincident counts behind a beam splitter. About half of the time, the two photons will choose the same output port of the beam splitter and no coincidence will be observed.

The average fidelity of the clones can be directly deduced from Fig. 2 by taking the ratio, R, between the maximum and base values in the $|2,0\rangle$ curves. The flatness of the $|1,1\rangle$ curves demonstrates that the observed peaks are indeed due to stimulation. From the discussion here, it follows that this is equal to the ratio between $p(2,0)$ and $p(1,1)$. Therefore, the relative probability for the two photons to have equal polarization is $R/(R+1)$, whereas the probability for them to have orthogonal polarizations is $1/(R+1)$. As a consequence, the average fidelity of the individual clones is

$$F = \frac{R}{R+1} \times 1 + \frac{1}{R+1} \times \frac{1}{2} = \frac{2R+1}{2R+2} \quad (5)$$

in analogy with Eq. (3). The observed values of R from Fig. 2 have uncertainties of the order of 3% and lead to values for the fidelity F of 0.81 ± 0.01, 0.80 ± 0.01, and 0.81 ± 0.01 for the three complementary polarization directions linear vertical, linear at 45°, and circular left-handed, respectively. The experimental values are close to the optimum value of 5/6 = 0.833 for a universal symmetric cloning machine. Strictly speaking, the clones are equally good only for perfect overlap. For imperfect overlap, one can in principle distinguish the input photon from the photon produced by down-conversion with a finite probability.

The absolute number of counts in Fig. 2 is determined by several factors: the pump pulse repetition rate (80 MHz), the probability for each input pulse to contain a photon (5×10^{-2}), the probability of producing a down-converted pair (10^{-3}), and the overall detection efficiency (0.10 per photon). Multiplication of all these factors leads to the observed levels.

The limiting factor for the quality of the clones in our experiment is the difference in (temporal) width between the input photons and the photons produced in the down-conversion process, leading to imperfect mode overlap. There are two reasons for this. First, the input photon goes through several additional optical elements that stretch the wave packet (Fig. 1). Second, the down-conversion process intrinsically has a shorter coherence time than the input pulse. This is largely compensated for with the use of 5-nm bandwidth interference filters in front of the detectors.

Another important practical point for the experiment is the compensation for the effects of birefringence, which is achieved by the three compensation crystals (Fig. 1). Birefringence leads to a time delay between vertical and horizontal polarization, which without compensation would considerably affect the overlap and thus the stimulating effect for 45° linear and circular polarizations. The fact that the stimulation effect for these polarizations is comparable to the vertical case (Fig. 2) indicates that the compensation is effective.

An interesting property of universal quantum cloning machines is that they constitute the optimal attack on certain quantum cryptography protocols (*15*). Applications of cloning in a quantum computing context were suggested in (*16*). From a more fundamental point of view, quantum cloning by stimulated emission shows how a basic quantum information procedure can be implemented in a natural way.

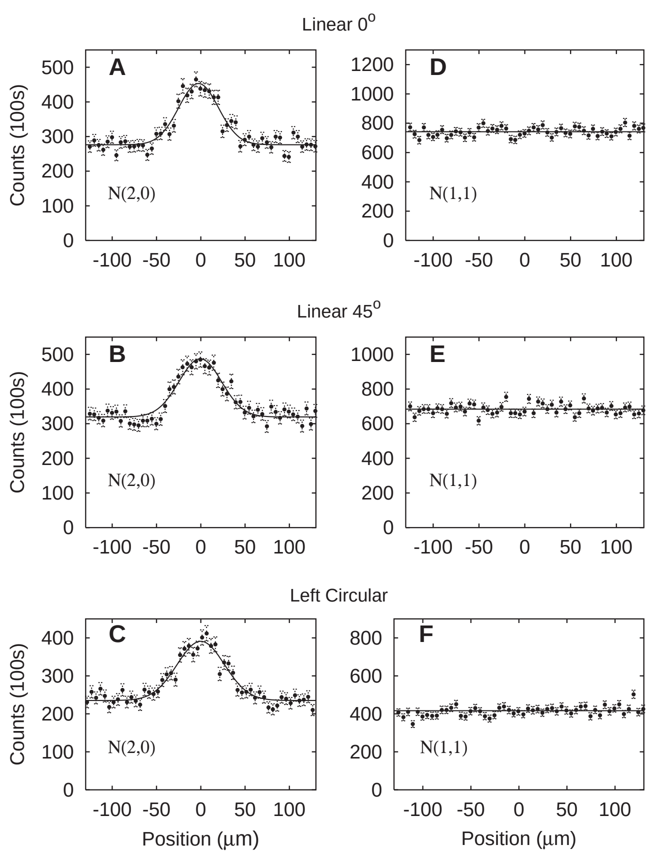

Fig. 2. (**A** to **C**) The number $N(2,0)$ of detections are shown where both photons in mode *a* have the polarization of the input photon. Input polarizations were linear vertical, linear at 45°, and circular left-handed, respectively. $N(2,0)$ is plotted versus the relative distance between input and produced photon. As expected, there is a marked increase in the overlap region. In the ideal case of perfect overlap, the increase would be by a factor of two. As required for universal cloning, the enhancement is similar for each input state. The polarization states chosen belong to three complementary bases, corresponding to the *x*, *y*, and *z* directions, for spin. Intermediate initial polarizations give similar results. (**D** to **F**) The number $N(1,1)$of detections are shown where the two photons have opposite polarization, for each of the same three input polarizations. As expected, $N(1,1)$ does not show any enhancement in the overlap region. The variation of the base rates for the different inputs is a consequence of variations in the pump power and the changes in optical elements between the different analyzer configurations.

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