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Parametric down-conversion with coherent pulse pumping and quantum interference between independent fields

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Abstract. The process of parametric down-conversion pumped by a short coherent pulse is analysed. The coherence properties of the down-converted field are investigated. It is found that the spectrum of the down-conversion is mostly determined by the nonlinear medium and is independent of the width of the short pumping pulse and that it is impossible to realize a transform-limited two-photon state. Fourth-order interference between a weak coherent pulse and the down-converted pulse is considered. It is found that high visibility near 100% is achievable by means of narrow optical filtering. It is shown that by gating the detection on one of the down-converted photons, a transform-limited single-photon pulse is achievable in the other conjugate field, but its pulse width is shorter than that of the down-converted pulse and its peak position is uncertain within the pulse width of the down-converted light. However, with narrow optical filtering on the gating detection process, a transform-limited single-photon state can be obtained in the gated field with its peak position synchronized to the pump pulse. The possibility for the realization of a multi-photon interference experiment is explored for the quantum state measurement of the single-photon state.

1. Introduction

Quantum interference with a pair of correlated photons has played an important role in the recent developments in the fundamental study of quantum nonlocality [1–5]. Even though more dramatic nonlocality violation involves more than two particles [6–8], new violations of locality by quantum mechanics are still being discovered that are associated with a pair of photons [9, 10]. Moreover, the two-photon interference effect has recently been employed to demonstrate an ideal quantum eraser [11, 12]. On the one hand, most of the two-photon sources employed in quantum interference come from the process of parametric down-conversion. This has limited its application to mostly quantum interference [13] and locality violation [14]. On the other hand, in the newly emerging field of quantum information processing [15, 16], quantum state manipulation depends on quantum superposition among different sources, which leads to quantum interference between independent fields. Furthermore, some of the new dramatic violations of locality by a single photon also require quantum interference between independent quantum sources [8, 17, 18].

As a matter of fact, the first experiment in fourth-order interference was performed with two independent coherent sources as early as in 1967 [19]. However, since classical sources were used, the visibility of the interference was limited to less than 50% [20], which makes it useless for quantum information processing. Detailed quantum theory for interference

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with independent sources was not available until 1988 [20]. But because of the success in quantum interference with correlated sources, this issue was set aside until recently.

Several proposals have been made to investigate quantum interference between two independent sources. Among them are the demonstration of nonlocal phase correlation in a parametric down-conversion process [21], nonlocality of a single-photon state [17, 18], quantum state teleportation [22] and the realization of multi-photon entangled states [8]. These phenomena rely on interference between a quantum state and a classical coherent state or between two independent quantum states. High visibility of the interference pattern is required in these phenomena. Another interesting quantum interference effect between two independent sources occurs when a single-photon state is homodyned with a strong coherent state of light, where multi-photon interference is exhibited in the photon number statistics [23, 24].

It has been shown [20] that visibilities as high as 100% can be achieved in fourth-order interference with independent quantum sources possessing sub-Poissonian photon statistics. On the one hand, the condition for high visibility requires that the response time of the photodetectors be much smaller than the intensity correlation time of the light sources involved. So far, the sole efficient source with sub-Poissonian photon statistics has been the process of parametric down-conversion. But the spontaneous parametric down-conversion process has a wide bandwidth of the order of 10^{13} Hz with a correlation time of less than 10^{-13} s, well below the response time ($\sim 10^{-10}$ s) of the best photodetectors with current technology. Of course, optical filtering can be applied to reduce the bandwidth but at the expense of a significant decrease in the signal level, though active filtering may overcome the problem of decreased signal level. On the other hand, the quantum theory in [20] for the interference between two independent sources was based on stationary fields (continuous wave). Rarity [25] recently proposed the use of a pulsed laser for precise timing of photodetection in order to avoid the problem of a slow detector. Under certain conditions, it was shown that near 100% visibility can be achieved in fourth-order interference between two independent sources of a single-photon state. However, in his analysis Rarity did not consider the phase-matching condition imposed on the pump frequency in the parametric down-conversion process. This phase-matching condition will substantially limit the performance of a short pumping pulse because the response from the nonlinear medium is slower than the fast process of the pump pulse.

For an interference effect involving nonstationary sources, we need to consider the temporal mode overlap between the two interference fields, as well as the overlap of their spatial distribution, in order to achieve high visibility [26]. Such a temporal overlap can be enforced by optical filters as in the treatment by Rarity. On the one hand, passive optical filtering can be modelled as a beamsplitter which may result in the reduction of the quantum state through the introduction of noise in the vacuum field and therefore change the quantum state of the field. On the other hand, since fourth-order interference depends on a coincidence measurement, the optical filter simply reduces the coincidence rate and is still a better technique for achieving high visibility. The question is then: what is the requirement on the filters for high visibility in fourth-order interference involving the parametric down-conversion process pumped by a short pulse?

It is known that a single-photon state can be realized by gated detection in parametric down-conversion [27]. With pulsed pumping, one would expect that the two-photon wavepacket will take the temporal profile of the pump and therefore produce a transform-limited two-photon pulse. However, because of the finite response bandwidth of the nonlinear medium due to the phase-matching condition, the down-converted fields will have a much wider temporal profile than the ultra-short pump pulse, as we will show in the following sections. In this case, can we still generate a transform-limited single-photon state by gated detection?

We will answer these questions in this paper. We start in section 2 by first treating systematically the process of parametric down-conversion pumped by a coherent pulse laser. This will provide us with the quantum state for the down-converted fields so that we can study their coherence properties. We will examine whether or not the down-converted fields are transform-limited. We then investigate in section 3 the condition under which high visibility in fourth-order interference between independent sources can be achieved. The problem of multi-photon interference with pulsed sources will be discussed in section 4. We will show how to obtain a transform-limited single-photon state.

2. The process of parametric down-conversion with pulsed pumping

The process of parametric down-conversion has been studied by a number of researchers with different emphasis [28, 29]. However, almost all of them assumed a CW pumping field, which is usually monochromatic. Even though in some treatments, the bandwidth of the pumping is nonzero, no coherence among the frequency components is assumed. Thus the pump field is a stationary field. For a nonstationary pumping pulse, the field amplitude can be decomposed into its Fourier components in the following form:

$$E_p(t) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega\, a_p(\omega) \mathrm{e}^{-\mathrm{j}\omega(t-t_p)} \tag{1}$$

where $a_p(\omega)$ has a definite phase relation among different ω and is a slowly changing function of ω (slower than $\mathrm{e}^{\mathrm{j}\omega t_p}$). Let the spectrum of the pump field be centred at ω_p and its bandwidth be denoted by $\Delta\omega_p$. Then the pump pulse has a temporal width that is the reciprocal bandwidth of $a(\omega)$: $\Delta T_p = 1/\Delta\omega_p$ and is centred at $t = t_p$. Let us further assume that the pump field propagates in a fixed direction denoted by the unit vector $\vec{\kappa}_p$. In the parametric down-conversion process, the quantum fluctuation of the pump field does not influence the outcome, so we treat it as a classical field here.

By starting with the interaction Hamiltonian for the parametric process and following the steps in [29], we may easily derive the state for the generated fields as

$$|\Phi\rangle = \int \mathrm{d}^3k_s\, \mathrm{d}^3k_i\, \mathrm{d}\omega_p\, a_p(\omega_p) \mathrm{e}^{\mathrm{j}\omega_p t_p} \Phi(\omega_p, \vec{k}_s, \vec{k}_i) \delta(\omega_p - \omega_s - \omega_i) \hat{a}^\dagger_{\vec{k}_s} \hat{a}^\dagger_{\vec{k}_i} |\mathrm{vac}\rangle \tag{2}$$

where

$$\Phi(\omega_p, \vec{k}_s, \vec{k}_i) = C(\omega_s, \omega_i) \prod_{m=1}^{3} \mathrm{sinc}[(\vec{k}_p - \vec{k}_s - \vec{k}_i)_m l_m/2] \tag{3}$$

with

$$\vec{k}_j = \frac{n_j(\omega_j)}{c} \omega_j \vec{\kappa}_j \qquad (j = s, i, p). \tag{4}$$

Here we denote the pump field by p and the two down-converted fields by s and i. $\vec{\kappa}_j$ is the unit vector of propagation for the corresponding fields. l_m $(m = 1, 2, 3)$ is the interaction length of the nonlinear medium. $C(\omega_s, \omega_i)$ is a slowly changing normalization function of ω_s, ω_i. The sinc function in Φ gives rise to the phase-matching condition for the down-converted fields. In the derivation we have also assumed that the system (nonlinear medium and pump laser) has been ready for a long time so that the interaction time can be taken to be infinity. This gives rise to the δ-function in Φ for the frequencies of the fields to guarantee the energy conservation at single-photon level.

For simplicity, let us only consider those down-converted fields that propagate in the same direction as the pump field. Then the quantum state can be simplified as

$$|\Phi\rangle = \int \mathrm{d}\omega_p\,\mathrm{d}\omega_s\,\mathrm{d}\omega_i\, a_p(\omega_p)\mathrm{e}^{\mathrm{j}\omega_p t_p}\Phi(\omega_p,\omega_s,\omega_i)\delta(\omega_p-\omega_s-\omega_i)\hat{a}_s^\dagger(\omega_s)\hat{a}_i^\dagger(\omega_i)|\mathrm{vac}\rangle \tag{5}$$

with

$$\Phi(\omega_p,\omega_s,\omega_i) = C(\omega_s,\omega_i)\,\mathrm{sinc}(\Delta k l/2) \tag{6}$$

where $\Delta k \equiv k_p - k_s - k_i$ is the phase mis-matching and l is the length of the nonlinear medium. The function Φ will determine the response of the nonlinear medium to the pulsed excitation by the pump field. Its bandwidth is mostly determined by the phase-matching condition that is governed by the sinc function of Φ. Assume that the phase-matching condition is satisfied for the down-converted fields at frequencies ω_{s0} and ω_{i0} and for the pump field at the centre frequency ω_{p0}, that is,

$$k_p(\omega_{p0}) - k_s(\omega_{s0}) - k_i(\omega_{i0}) = 0. \tag{7}$$

Set $\omega_p = \omega_{p0} + \Omega_p$ and $\omega_s = \omega_{s0} + \Omega_p/2 + \Omega$ in equation (5). Then we have from the energy conservation condition (the δ-function in equation (5)):

$$\omega_i = \omega_{i0} + \Omega_p/2 - \Omega \tag{8}$$

and equation (5) becomes

$$|\Phi\rangle = \int \mathrm{d}\Omega_p\,\mathrm{d}\Omega\, a_p(\Omega_p)\mathrm{e}^{\mathrm{j}\Omega_p t_p}\Phi(\Omega_p,\Omega)\hat{a}_s^\dagger(\Omega_p/2+\Omega)\hat{a}_i^\dagger(\Omega_p/2-\Omega)|\mathrm{vac}\rangle \tag{9}$$

with

$$\Phi(\Omega_p,\Omega) = C(\Omega_p/2+\Omega,\Omega_p/2-\Omega)\,\mathrm{sinc}[\Delta k(\Omega_p,\Omega)l/2] \tag{10}$$

where we omit the centre frequencies for brevity. By expanding around phase-matching frequencies, $\Delta k(\Omega_p,\Omega)$ can be written as

$$\begin{aligned}\Delta k(\Omega_p,\Omega) &\equiv k_p(\omega_p) - k_s(\omega_s) - k_i(\omega_i)\\ &= \big(k_p' - \tfrac{1}{2}(k_i'+k_s')\big)\Omega_p + (k_i'-k_s')\Omega + \tfrac{1}{2}\big(k_p'' - \tfrac{1}{4}(k_i''+k_s'')\big)\Omega_p^2\\ &\quad + \tfrac{1}{2}(k_i''-k_s'')\Omega_p\Omega - \tfrac{1}{2}(k_s''+k_i'')\Omega^2\end{aligned} \tag{11}$$

where $k_m' = \mathrm{d}k_m/\mathrm{d}\omega_m, k_m'' = \mathrm{d}^2k_m/\mathrm{d}\omega_m^2$ $(m = s, i, p)$. Since we will discuss interference between independent fields, the centre frequency for the two down-converted fields is preferred to be the same. So we will be mostly interested in the degenerate case where $\omega_{s0} = \omega_{i0} = \omega_{p0}/2$. Furthermore, for the degenerate type I phase matching, we have $k_s' = k_i'$ and $k_s'' = k_i''$. Thus equation (11) becomes

$$\Delta k(\Omega_p,\Omega) = (k_p' - k_s')\Omega_p + (2k_p'' - k_s'')\Omega_p^2/4 - k_s''\Omega^2. \tag{12}$$

The bandwidth of the function $\Phi(\Omega_p,\Omega)$ is then deduced from the sinc function in equation (11) by using the following relation:

$$|\Delta k(\Omega_p,\Omega)|l \leqslant \pi/2 \tag{13a}$$

or

$$|(k_p'-k_s')\Omega_p + (2k_p''-k_s'')\Omega_p^2/4 - k_s''\Omega^2|l \leqslant \pi/2. \tag{13b}$$

Thus the bandwidth of Φ as a function of Ω_p is determined by $\Delta\Omega_p \equiv \pi/2l|(k_p'-k_s')|$, while that for Ω is given by $\Delta\Omega \equiv \sqrt{\pi/2lk_s''}$. Since $\Delta\Omega$ arises from the second-order term and $\Delta\Omega_p$ from the first-order term in equation (12), we usually have $\Delta\Omega_p \ll \Delta\Omega$. Even though $k_s' \neq k_i'$ for type II phase-matched down-conversion so that $\Delta\Omega$ also comes from the first-order term in equation (11), the dispersion in a nonlinear medium will make $|k'_p - (k'_i + k'_s)/2| \gg |k'_s - k'_i|$ and we still have $\Delta\Omega_p \ll \Delta\Omega = \pi/2l|(k'_i - k'_s)|$. In the remainder of the paper, we assume that the pump pulse is very short so that the pump spectrum is wider than both $\Delta\Omega_p$ and $\Delta\Omega$, that is, $\Delta\omega_p \gg \Delta\Omega_p$. Then $a_p(\Omega_p)$ is a slowly-varying quantity compared with $\Phi(\Omega_p, \Omega)$ and does not play a role in the integral in equation (9). Thus the effect of the ultra-short pulse will be limited by $\Delta\Omega_p$, which can be thought of as the equivalent pumping bandwidth. $\Delta\Omega$ is the intrinsic down-conversion bandwidth which corresponds to the case with a monochromatic pump field. The overall bandwidth of the down-converted fields with a pulsed pump is determined by equation (9) as a convolution between the bandwidths of $\Delta\Omega_p$ and $\Delta\Omega$.

Let us now examine the coherence properties of the down-converted fields. We first find the intensity $\langle \hat{I}_{s,i}(t)\rangle = \langle \hat{E}^{(-)}_{s,i}(t)\hat{E}^{(+)}_{s,i}(t)\rangle$, where $\hat{E}^{(+)}_{s,i}(t) = \int d\omega\, \hat{a}_{s,i}(\omega)e^{-j\omega t}/\sqrt{2\pi}$ are the field operators for the down-converted fields. It is straightforward to show that

$$\langle \hat{I}_s(t)\rangle = \langle \hat{I}_i(t)\rangle = \int d\Omega'\, |A_p(t, \Omega')|^2 \tag{14}$$

with

$$A_p(t, \Omega') \equiv \frac{1}{\sqrt{2\pi}} \int d\Omega_p\, a_p(\Omega_p)\Phi(\Omega_p, \Omega_p/2 + \Omega')e^{-j(\Omega_p+\Omega')(t-t_p)}. \tag{15}$$

Obviously, the intensity of the down-converted fields is time dependent. It forms a pulse centred at $t = t_p$ or the centre of the pump pulse. The temporal width of the pulse is determined by that of $A_p(t, \Omega')$. For a short pump pulse with $\Delta\omega_p \gg \Delta\Omega_p, \Delta\Omega$, the temporal width for the down-converted field can be derived from equation (15) as

$$\Delta T_{s,i} = \max(1/\Delta\Omega_p, 1/\Delta\Omega). \tag{16}$$

On the other hand, we may rearrange equation (9) as follows:

$$|\Phi\rangle = \int d\Omega_1\, d\Omega_2\, a_p(\Omega_1 + \Omega_2)e^{j\Omega_p t_p}\Phi(\Omega_1 + \Omega_2, (\Omega_1 - \Omega_2)/2)\hat{a}^\dagger_s(\Omega_1)\hat{a}^\dagger_i(\Omega_2)|\text{vac}\rangle. \tag{17}$$

Hence, the bandwidth for the down-converted fields is determined by the range of Ω_1 or Ω_2 allowed in equation (17). From equation (13), we find

$$\Omega_1 + \Omega_2 < \Delta\Omega_p \qquad |\Omega_1 - \Omega_2| < \Delta\Omega \tag{18a}$$

which results in the range for Ω_1, Ω_2:

$$|\Omega_1|,\ |\Omega_2| \sim (\Delta\Omega_p + \Delta\Omega)/2. \tag{18b}$$

Hence the bandwidth for Ω_1, Ω_2 is

$$\Delta\Omega_s, \Delta\Omega_i \sim (\Delta\Omega_p + \Delta\Omega). \tag{18c}$$

Thus the bandwidth of the down-conversion is determined by the sum of $\Delta\Omega_p$ for the medium response bandwidth and $\Delta\Omega$ of the intrinsic down-conversion bandwidth.

Combining equations (16) and (18*c*), we may conclude that the down-converted fields will never be transform-limited because we always have $\Delta\Omega_{s,i} > 1/\Delta T_{s,i}$, or in other words, we can never obtain a transform-limited two-photon state from the down-conversion process.

This conclusion can be further confirmed by considering the correlation function

$$\gamma(t_1, t_2) \equiv \frac{|\langle \hat{E}^{(-)}(t_2)\hat{E}^{(+)}(t_1)\rangle|^2}{\langle \hat{I}(t_1)\rangle\langle \hat{I}(t_2)\rangle}. \tag{19}$$

The necessary and sufficient condition for an optical field to be transform limited is simply $\gamma(t_1, t_2) = 1$ for all t_1, t_2. It is straightforward to show by using equation (9) as the quantum state

$$\langle \hat{E}_s^{(-)}(t_2)\hat{E}_s^{(+)}(t_1)\rangle = \int d\Omega' \, A_p(t_1, \Omega')A_p^*(t_2, \Omega') \tag{20}$$

where $A_p(t, \Omega')$ is given in equation (15). By using the Schwartz inequality, we have

$$\left| \int d\Omega' \, A_p(t_1, \Omega')A_p^*(t_2, \Omega') \right|^2 \leqslant \int d\Omega' \, |A_p(t_1, \Omega')|^2 \int d\Omega' \, |A_p(t_2, \Omega')|^2. \tag{21}$$

Combining equations (14), (20) and (21), we have

$$|\langle \hat{E}_s^{(-)}(t_2)\hat{E}_s^{(+)}(t_1)\rangle|^2 \leqslant \langle \hat{I}_s(t_1)\rangle\langle \hat{I}_s(t_2)\rangle \qquad \text{or} \qquad \gamma(t_1, t_2) \leqslant 1. \tag{22}$$

The equality sign in the above relation stands if and only if

$$A_p(t_1, \Omega') = C A_p(t_2, \Omega') \qquad \text{with} \quad C = C(t_1, t_2). \tag{23}$$

C is independent of Ω'. An examination of equation (15) with (10) for $\Phi(\Omega_p, \Omega)$ shows that it is impossible to satisfy equation (23). Therefore, $\gamma(t_1, t_2) < 1$ for $t_1 \neq t_2$ and the signal and idler fields can never be transform limited.

This fact may influence the temporal mode match in the quantum interference between independent sources.

3. Fourth-order interference between independent nonstationary sources

Fourth-order interference involves coincidence measurement of two photons. For interference between two independent sources, it is known [20] that in order to achieve high visibility, at least one of the fields must have sub-Poissonian photon statistics. A single-photon state can be produced from the process of parametric down-conversion by gating the detection of one of the down-converted fields on the detection of the other conjugate field [27]. So let us consider interference between coherent pulses and the signal field from down-conversion. As shown in figure 1, the signal field is first divided by a 50:50 beamsplitter. The divided beams are then mixed, respectively, with two coherent pulses by beamsplitters $BS_{1,2}$. A coincidence measurement on the two output fields of the beamsplitters $BS_{1,2}$ is performed. In order to obtain a single-photon state for the signal field, we gate the coincidence measurement on the detection of the idler field. Such a scheme was first proposed by Tan *et al* to demonstrate nonlocality of a single photon [17].

Let us denote the field operator for the coherent pulses by $\hat{\mathcal{E}}_{1,2}^{(+)}(t)$. Assume that the coherent pulses propagate in a fixed direction. Then the field operator can be expressed in terms of the annihilation operator $\hat{b}_{1,2}(\omega)$ as

$$\hat{\mathcal{E}}_{1,2}^{(+)}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \, \hat{b}_{1,2}(\omega) e^{-j\omega t}. \tag{24}$$

For coherent pulses, the quantum states $|B\rangle_{1,2}$ can be described by the following relation:

$$\hat{b}_{1,2}(\omega)|B\rangle_{1,2} = \beta_{1,2}(\omega) e^{j\omega t_p} |B\rangle_{1,2} \tag{25}$$

where $\beta_{1,2}(\omega)$ are slowly varying functions of ω with a definite phase relation for different ω in order to form coherent pulses, the widths of which are determined by the reciprocal of the bandwidth $\Delta\omega_{1,2}$ of $\beta_{1,2}(\omega)$. The coherent pulses are centred at $t = t_p$ and therefore are synchronized to the pump pulse.

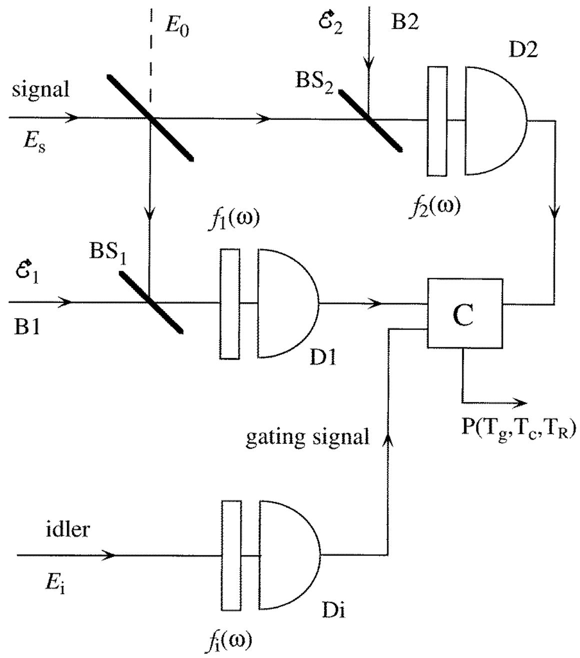

Figure 1. Fourth-order interference between the signal field of the down-conversion and coherent fields gated by the detection of the idler field.

The output fields of the two beamsplitters can be expressed as

$$\begin{aligned}\hat{E}_1^{(+)}(t) &= \frac{1}{\sqrt{2}}\left\{\hat{\mathcal{E}}_1^{(+)}(t) + \frac{\mathrm{j}}{\sqrt{2}}\left[\hat{E}_s^{(+)}(t) + \mathrm{j}\hat{E}_0^{(+)}(t)\right]\right\} \\ \hat{E}_2^{(+)}(t) &= \frac{1}{\sqrt{2}}\left\{\hat{\mathcal{E}}_1^{(+)}(t) + \frac{\mathrm{j}}{\sqrt{2}}\left[\mathrm{j}\hat{E}_s^{(+)}(t) + \hat{E}_0^{(+)}(t)\right]\right\}\end{aligned} \tag{26}$$

where $\hat{E}_0^{(+)}(t) = \int \mathrm{d}\omega\, \hat{a}_0(\omega)\mathrm{e}^{-\mathrm{j}\omega t}/\sqrt{2\pi}$ is the field operator for the vacuum input port of the first beamsplitter. In order to enforce temporal mode overlap between the two interfering fields for high visibility in the interference, we put optical filters in front of the detectors. Let us denote the transmission of the filters by $f_1(\omega)$, $f_2(\omega)$, $f_i(\omega)$, which are all centred at $\omega_{p0}/2$ and have a bandwidth of $\Delta\Omega_{f1,f2,fi}$. Then the field operators at the detectors have the form of

$$\begin{aligned}\hat{E}_{\mathrm{D}1}^{(+)}(t) &= \int \mathrm{d}\omega \frac{1}{\sqrt{4\pi}}\left\{\left[\hat{b}_1(\omega) + \frac{\mathrm{j}}{\sqrt{2}}[\hat{a}_s(\omega) + \mathrm{j}\hat{a}_0(\omega)\right]\right\} f_1(\omega)\mathrm{e}^{-\mathrm{j}\omega t} \\ \hat{E}_{\mathrm{D}2}^{(+)}(t) &= \int \mathrm{d}\omega \frac{1}{\sqrt{4\pi}}\left\{\left[\hat{b}_2(\omega) + \frac{\mathrm{j}}{\sqrt{2}}[j\hat{a}_s(\omega) + \hat{a}_0(\omega)\right]\right\} f_2(\omega)\mathrm{e}^{-\mathrm{j}\omega t} \\ \hat{E}_{\mathrm{D}i}^{(+)}(t) &= \frac{1}{\sqrt{2\pi}}\int \mathrm{d}\omega\, \hat{a}_i(\omega) f_i(\omega)\mathrm{e}^{-\mathrm{j}\omega t}.\end{aligned} \tag{27}$$

The coincidence rate between D1 and D2 gated on the detection of the idler field is then proportional to the quantity

$$P(T_c, T_g, T_R) = \int_{T_R} \mathrm{d}t \int_{T_c} \mathrm{d}\tau_2 \int_{T_g} \mathrm{d}\tau_1\, G(t, t+\tau_1, t+\tau_1+\tau_2) \tag{28}$$

with

$$G(t_1, t_2, t_3) = \langle \Phi, B|\hat{E}^{(-)}_{\mathrm{D}i}(t_1)\hat{E}^{(-)}_{\mathrm{D2}}(t_2)\hat{E}^{(-)}_{\mathrm{D1}}(t_3)\hat{E}^{(+)}_{\mathrm{D1}}(t_3)\hat{E}^{(+)}_{\mathrm{D2}}(t_2)\hat{E}^{(+)}_{\mathrm{D}i}(t_1)|\Phi, B\rangle. \quad (29)$$

Here T_g is the gating time after the detection of a photon at Di, T_c is the coincidence time between D1 and D2, and T_R is the resolution time for detector Di. Normally, $T_{g,c}$ are of the same order as the resolution time T_R.

With the states $|\Phi, B\rangle$ given in equations (9) and (25), it is straightforward to calculate the quantity $G(t_1, t_2, t_3)$. After some lengthy derivation, we arrive at

$$G(t_1, t_2, t_3) = \tfrac{1}{8}|B_1(t_3)g_2(t_1, t_2) + B_2(t_2)g_1(t_1, t_3)|^2 + \tfrac{1}{4}|B_1(t_2)B_2(t_3)|^2 I_{\mathrm{D}i}(t_1) \quad (30a)$$

with

$$g_k(t, t') = \frac{1}{2\pi}\int \mathrm{d}\Omega\, \mathrm{d}\Omega_p\, a_p(\Omega_p)\mathrm{e}^{\mathrm{j}\Omega_p t_p}\Phi(\Omega_p, \Omega) f_k(\Omega_p/2 + \Omega) \times f_i(\Omega_p/2 - \Omega)\mathrm{e}^{-\mathrm{j}(\Omega_p/2+\Omega)t'}\mathrm{e}^{-\mathrm{j}(\Omega_p/2-\Omega)t} \qquad (k = 1, 2) \quad (30b)$$

and

$$B_k(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}\omega\, f_k(\omega)\beta_k(\omega)\mathrm{e}^{-\mathrm{j}\omega(t-t_p)} \qquad (k = 1, 2). \quad (30c)$$

$I_{\mathrm{D}i}(t_1)$ is the intensity of the idler field after the filter. The last term in equation (30a) is the accidental coincidence which comes from the coherent pulse while the first term corresponds to interference. In order to have close to 100% visibility for interference, it is desirable to make the coherent pulses as weak as possible so that we can omit the last term which is in a quadratic form of $|B_{1,2}(t)|^2$ [17, 20]. Then we have

$$G(t_1, t_2, t_3) = \tfrac{1}{4}|B_1(t_3)g_2(t_1, t_2) + B_2(t_2)g_1(t_1, t_3)|^2. \quad (31)$$

An interference fringe will show up in the gated coincidence count $P(T_g, T_c, T_R)$ as a function of the phase difference between B_1 and B_2.

Obviously, if $T_g, T_c, T_R \ll 1/\Delta\Omega, 1/\Delta\Omega_p, \Delta\omega, 1/\Delta\Omega_f$, we will have

$$P(T_g, T_c, T_R) = G(0, 0, 0)T_g T_c T_R = \tfrac{1}{4}T_g T_c T_R|B_1(0)g_2(0, 0) + B_2(0)g_1(0, 0)|^2. \quad (32)$$

Hence we will always obtain 100% visibility for the interference fringe if we can make $|B_1(0)g_2(0, 0)| = |B_2(0)g_1(0, 0)|$ by adjusting the ratio between the intensities of the coherent pulses. The condition on T_g, T_c, T_R requires that the response of the detectors be much faster than the fluctuations of the fields. This is exactly what was derived in [20] for stationary fields. On the other hand, if $T_g, T_c, T_R \gg 1/\Delta\Omega, 1/\Delta\Omega_p, 1/\Delta\Omega_f$, the integration in equation (28) is over a range that is wider than that of the functions $g_{1,2}(t, t')$ and $B_{1,2}(t)$. In order to achieve 100% visibility in the interference pattern, we require the temporal shapes of $B_{1,2}(t)$ and $g_{1,2}(t, t')$ be matched. This is the so-called temporal mode match [27]. Because of the complexity in the function $\Phi(\Omega_p, \Omega)$, it is hard to match $B_{1,2}(t)$ and $g_{1,2}(t, t')$ without the aid of the filters. The role of the filters is to enforce the matching of the temporal shapes of the coherent pulses and the down-converted fields. Let us now examine $B_{1,2}(t)$ and $g_{1,2}(t, t')$ more closely by setting $\Omega_1 = \Omega_p/2+\Omega$ and $\Omega_2 = \Omega_p/2-\Omega$ in equation (30b). Then we have

$$g_{1,2}(t, t') = \int \mathrm{d}\Omega_1\, \mathrm{d}\Omega_2\, a_p(\Omega_1 + \Omega_2)\Phi(\Omega_1 + \Omega_2, (\Omega_1 - \Omega_2)/2) \times f_{1,2}(\Omega_1) f_i(\Omega_2)\mathrm{e}^{-\mathrm{j}\Omega_1(t'-t_p)}\mathrm{e}^{-\mathrm{j}\Omega_2(t-t_p)}. \quad (33)$$

The only way that $g_{1,2}(t, t')$ can be matched to $B_{1,2}(t)$ is when $f_1(\Omega) = f_2(\Omega) \equiv f(\Omega)$ and the bandwidths of the filters (f_1, f_2, f_i) are much smaller than the bandwidths $\Delta\Omega_p$, $\Delta\Omega$ and $\Delta\omega$, or

$$\{\Delta\Omega_{f1}, \Delta\Omega_{f2}, \Delta\Omega_{fi}\} \ll \{\Delta\Omega_p, \Delta\Omega, \Delta\omega\} \tag{34}$$

so that the pulse widths for both $B_{1,2}(t)$ and $g_{1,2}(t, t')$ are determined by the bandwidth of $f(\Omega)$. Under this condition, we may make the approximation by replacing $a_p(\Omega_1 + \Omega_2)$ with $a_p(0)$ and $\Phi(\Omega_1 + \Omega_2, (\Omega_1 - \Omega_2)/2)$ with $\Phi(0, 0)$ in the integration in equation (33) and $\beta_{1,2}(\omega)$ with $\beta_{1,2}(0)$ in equation (30*c*). Thus we have

$$\begin{aligned} g(t_1, t_2, t_3) &\approx a_p(0)\Phi(0,0) \int \mathrm{d}\Omega_1\, \mathrm{d}\Omega_2\, f(\omega) f(\Omega_1) f_i(\Omega_2) \mathrm{e}^{-\mathrm{j}\Omega_1(t-t_p)} \mathrm{e}^{-\mathrm{j}\Omega_2(t'-t_p)} \\ &= a_p(0)\Phi(0,0) F(t) F_i(t') \end{aligned} \tag{35a}$$

and

$$B_{1,2}(t) = \beta_{1,2}(0) F(t). \tag{35b}$$

where $F(t) = \int \mathrm{d}\Omega\, f(\Omega)\mathrm{e}^{-\mathrm{j}\Omega(t-t_p)}/\sqrt{2\pi}$ and $F_i(t) = \int \mathrm{d}\Omega\, f_i(\Omega)\mathrm{e}^{-\mathrm{j}\Omega(t-t_p)}/\sqrt{2\pi}$. Then the coincidence rate becomes

$$P(T_g, T_c, T_R) = \tfrac{1}{4}|a_p(0)\Phi(0,0)|^2 |\beta_1(0) + \beta_2(0)|^2 \left\{ \int \mathrm{d}t_1\, |F(t_1)|^2 \right\}^2 \int \mathrm{d}t_2\, |F_i(t_2)|^2. \tag{36}$$

Evidently, we can have 100% visibility in the interference if the strength of the two coherent pulses is adjusted to be equal.

4. Multi-photon interference between a single-photon state and a strong coherent state

When a single-photon state is mixed with a coherent state by a 50:50 beamsplitter, an interesting multi-particle interference effect will occur: the output photon statistics of the beamsplitter will not be the simple binomial distribution as expected from classical particle theory even if the coherent state is very strong; quantum multi-particle interference will cause the probability to drop to zero for equal output photon numbers, for which the simple classical binomial distribution would produce maximum probability [23, 24]. Such an effect is a manifestation of the quantum mechanical behaviour of a single-photon state. In a stationary case, the observation of this effect requires exact matching of the bandwidth of the single-photon field with that of the detector response [30], which is really equivalent to the overlap of the temporal modes for the two interference fields in the nonstationary case. In the following, we will consider the effect for the case of a pulsed single-photon state and a pulsed coherent state and explore the possibility of observing this effect with a down-converted field as the single-photon state.

First of all, let us consider a nonstationary single-photon state described by

$$|\Psi\rangle = \int \mathrm{d}\omega\, \Psi(\omega) \mathrm{e}^{\mathrm{j}\omega t_0} \hat{a}^\dagger(\omega)|\mathrm{vac}\rangle \tag{37a}$$

with normalization condition

$$\int \mathrm{d}\omega\, |\Psi(\omega)|^2 = 1. \tag{37b}$$

It can be easily shown that the intensity of the field in this state has the form of

$$I(t) = \langle \hat{E}^{(-)}(t) \hat{E}^{(+)}(t) \rangle = |A(t)|^2 \tag{38}$$

with

$$A(t) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega\, \Psi(\omega) \mathrm{e}^{-\mathrm{j}\omega(t-t_0)}. \tag{39}$$

It can be easily checked that the quantity $\gamma(t_1, t_2) = 1$ for the state in equation (37). Thus, it is a transform-limited single-photon pulse centred at $t = t_0$. It is straightforward to show from equation (37*b*) that the total photon number is

$$\int_{-\infty}^{\infty} \mathrm{d}\tau\, I(\tau) = \int_{-\infty}^{\infty} \mathrm{d}\tau\, |A(\tau)|^2 = 1. \tag{40}$$

Next, we mix this single-photon field with a strong coherent pulse by a 50:50 beamsplitter as shown in figure 2. The coherent pulse is described by equation (25) with $\beta_1(\omega) = \beta(\omega)\mathrm{e}^{\mathrm{j}\omega T_0}$. The intensity of the coherent pulse is given by

$$I_{\mathcal{E}}(t) = \langle \hat{\mathcal{E}}^{(-)}(t) \hat{\mathcal{E}}^{(+)}(t) \rangle = |B(t)|^2 \tag{41}$$

with

$$B(t) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega\, \beta(\omega) \mathrm{e}^{-\mathrm{j}\omega(t-T_0)}. \tag{42}$$

So the pulse is also transform limited and is centred at $t = T_0$. The output fields of the beamsplitter are described by the field operators:

$$\begin{aligned} \hat{E}_1^{(+)}(t) &= \frac{1}{\sqrt{2}} \big[\hat{E}^{(+)}(t) + \mathrm{j}\hat{\mathcal{E}}^{(+)}(t) \big] \\ \hat{E}_2^{(+)}(t) &= \frac{1}{\sqrt{2}} \big[\mathrm{j}\hat{E}^{(+)}(t) + \hat{\mathcal{E}}^{(+)}(t) \big]. \end{aligned} \tag{43}$$

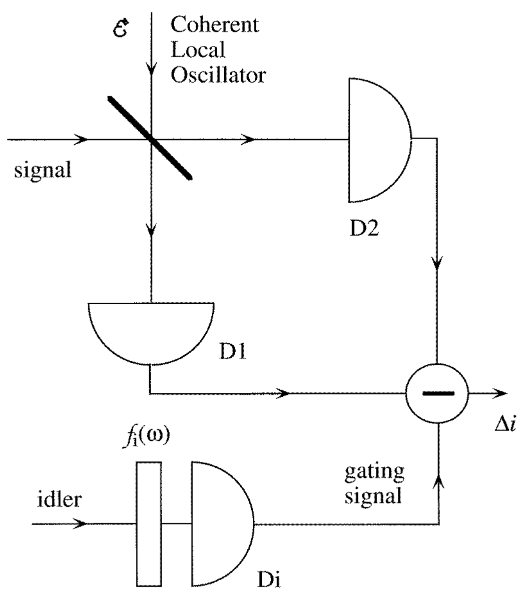

Figure 2. Scheme for multi-photon interference involving a single-photon state and a strong coherent state. The single-photon state is produced in the signal field of the down-conversion by gated detection upon the detection of the idler field.

We will be interested in the probability distribution for the photon number difference between the two output ports. It can be calculated from the joint probability $P_{N_1N_2}$ of finding N_1 photons in port 1 and N_2 photons in port 2, which is given by

$$P_{N_1N_2} = \left\langle \mathcal{T} : \frac{\hat{W}_1^{N_1}(T)}{N_1!} \mathrm{e}^{-\hat{W}_1(T)} \frac{\hat{W}_2^{N_2}(T)}{N_2!} \mathrm{e}^{-\hat{W}_2(T)} : \right\rangle \tag{44}$$

with $\hat{W}_{1,2}(T) = \int_0^T \mathrm{d}\tau\, \hat{I}_{1,2}(\tau)$. However, this quantity is not easy to calculate for the multimode states in equations (25) and (37). On the other hand, photodetectors make a quantum measurement of the photon number of optical fields, that is, the photoelectrons have the same statistics as the photons that fall on the detector [30]. So we can equivalently check the photocurrent fluctuations for the two detectors located at the outputs of the beamsplitter and calculate the probability distribution of the photocurrents. The general formula for the characteristic function of the photocurrent fluctuations has been calculated in [30]. Under the large intensity condition, it has the following form for two detectors (equation (44) of [30]):

$$\begin{aligned} C_{i_1,i_2}(r_1, r_2) &= \langle \exp\{\mathrm{j}r_1 i_1 + \mathrm{j}r_2 i_2\}\rangle \\ &\approx \left\langle \mathcal{T} : \exp\left\{ \mathrm{j}\alpha r_1 \int_0^\infty \mathrm{d}\tau\, \hat{I}_1(t-\tau) Q(\tau) + \mathrm{j}\alpha r_2 \int_0^\infty \mathrm{d}\tau\, \hat{I}_2(t-\tau) Q(\tau) \right.\right. \\ &\quad \left.\left. - \tfrac{1}{2}\alpha r_1^2 \int_0^\infty \mathrm{d}\tau\, \hat{I}_1(t-\tau)[Q(\tau)]^2 - \tfrac{1}{2}\alpha r_2^2 \int_0^\infty \mathrm{d}\tau\, \hat{I}_2(t-\tau)[Q(\tau)]^2 \right\} : \right\rangle \end{aligned} \tag{45}$$

where α and $Q(\tau)$ are the quantum efficiency and the response function for the two detectors, respectively. For simplicity, we assume that the two detectors are identical. In the detection of the nonstationary field, especially for ultra-fast laser pulses, the response of the detectors is usually slow so that their action is simply an average of the photocurrent over a long period of time T (longer than the optical pulse width). Hence we can choose the response function as

$$Q(\tau) = \begin{cases} i_0 & \text{for} \quad 0 < \tau < T \\ 0 & \text{for other } \tau \end{cases} \tag{46}$$

where i_0 is the average photocurrent in the detectors during the period T. For a balanced homodyne detection scheme as shown in figure 2, the photocurrents from the two detectors are subtracted to produce a difference current $\Delta i = i_1 - i_2$, which represents the photon number difference between the two outputs of the beamsplitter. We will find the probability distribution for the photocurrent difference Δi. Its characteristic function can be obtained from equation (45) by setting $r_1 = -r_2 \equiv r$. With equation (46) for $Q(\tau)$, we have

$$\begin{aligned} C_{\Delta i}(r) &= \langle \exp\{\mathrm{j}r\Delta i\}\rangle \\ &\approx \left\langle \mathcal{T} : \exp\left\{ \mathrm{j}r\alpha i_0 \int_0^T \mathrm{d}\tau \left[\hat{I}_1(t-\tau) - \hat{I}_2(t-\tau)\right] \right.\right. \\ &\quad \left.\left. - \tfrac{1}{2} r^2 \alpha i_0^2 \int_0^T \mathrm{d}\tau \left[\hat{I}_1(t-\tau) + \hat{I}_2(t-\tau)\right] \right\} : \right\rangle. \end{aligned} \tag{47}$$

From equation (43), we find

$$\begin{aligned} \hat{I}_1 - \hat{I}_2 &= \mathrm{j}\left[\hat{E}^{(-)}\hat{\mathcal{E}}^{(+)} - \hat{\mathcal{E}}^{(-)}\hat{E}^{(+)}\right] \\ \hat{I}_1 + \hat{I}_2 &= \hat{E}^{(-)}\hat{E}^{(+)} + \hat{\mathcal{E}}^{(-)}\hat{\mathcal{E}}^{(+)}. \end{aligned} \tag{48}$$

So equation (47) becomes

$$C_{\Delta i}(r) = \left\langle \mathcal{T} : \exp\left\{-r\alpha i_0 \int_0^T \mathrm{d}\tau \left[B(t-\tau)\hat{E}^{(-)} - B^*(t-\tau)\hat{E}^{(+)}\right] - \frac{r^2\alpha i_0^2}{2}\int_0^T \mathrm{d}\tau\, \hat{I}(t-\tau)\right\} : \right\rangle \exp\left[-\frac{r^2\alpha i_0^2}{2}\int_0^T \mathrm{d}\tau\, |B(t-\tau)|^2\right] \quad (49)$$

where we replace the operator $\hat{\mathcal{E}}^{(+)}$ with the c-function $B(t)$ for the coherent pulse because of the normal ordering : :. It is now straightforward to calculate $C_{\Delta i}(r)$. By expanding the exponential function inside the angled brackets in equation (49) and using equation (37) for the single-photon state, we find

$$C_{\Delta i}(r) = \left\{1 + \tfrac{1}{2}r^2\alpha^2 i_0^2 \left\langle : \left[\int_0^T \left[B(t-\tau)\hat{E}^{(-)} - B^*(t-\tau)\hat{E}^{(+)}\right]\mathrm{d}\tau\right]^2 : \right\rangle\right\} \times \exp\left[-\tfrac{1}{2}r^2\alpha i_0^2 \int_0^T \mathrm{d}\tau\, |B(t-\tau)|^2\right] \quad (50)$$

where higher-order terms in the expansion are zero due to the single-photon property of the state $|\Psi\rangle$ and we neglect the term with $\hat{I}(t)$ because we assume the coherent field to be much stronger than the single-photon field. With the state $|\Psi\rangle$ in equation (37), the quantity inside the angled brackets can be easily calculated and we have

$$C_{\Delta i}(r) = (1 - ar^2)\mathrm{e}^{-br^2/2} \quad (51)$$

with

$$\begin{aligned} a &\equiv \alpha^2 i_0^2 \left|\int_0^T \mathrm{d}\tau\, B^*(t-\tau)A(t-\tau)\right|^2 \\ b &\equiv \alpha i_0^2 \int_0^T \mathrm{d}\tau\, |B(t-\tau)|^2. \end{aligned} \quad (52)$$

The probability distribution for the photocurrent difference Δi can be easily obtained from the characteristic function in equation (51) by a Fourier transformation. Hence

$$\begin{aligned} P_{\Delta i}(x) &= \frac{1}{2\pi}\int \mathrm{d}r\, C_{\Delta i}(r)\mathrm{e}^{-\mathrm{j}rx} \\ &= \frac{1}{\sqrt{2\pi b}}\left(1 - \frac{a}{b} + \frac{ax^2}{b^2}\right)\mathrm{e}^{-x^2/2b}. \end{aligned} \quad (53)$$

The distribution $P_{\Delta I}(x)$ has a twin-peak feature with a minimum at $x = 0$. This is the multi-photon quantum interference effect when a single-photon state and a strong coherent state are superposed. When $a = b$, complete cancellation of probability is achieved at $x = 0$. However, by using the Schwartz inequality, we have

$$a \leqslant \alpha b \int_0^T \mathrm{d}\tau\, |A(t-\tau)|^2. \quad (54)$$

The equality occurs when $B(\tau) = CA(\tau)$ with C a constant number, that is, the shape of the coherent pulse matches exactly with that of the single-photon state. Because $\alpha \leqslant 1$ and

$$\int_0^T \mathrm{d}\tau\, |A(t-\tau)|^2 \leqslant \int_{-\infty}^{\infty} \mathrm{d}\tau\, |A(t-\tau)|^2 = 1$$

we always have $a \leqslant b$, which ensures the positiveness of the probability distribution $P_{\Delta i}(x)$ for arbitrary x. Therefore, in order to observe complete probability cancellation, we need a

100% quantum efficiency for the detectors and a long integration time and most importantly the overlap of the temporal modes of the two interfering fields. Of course, these are the ideal conditions, which can never be met in practice. Any imperfectness in the experimental set-up will result in $a/b < 1$, which can be equivalent to a less than 100% quantum efficiency for the detectors ($\alpha_e \equiv a/b$). Although no perfect cancellation can be achieved, there is still a minimum at $x = 0$ unless $\alpha_e \leqslant \frac{1}{3} \approx 33.3\%$, for which the twin peak in $P_{\Delta I}(x)$ will merge into a single peak and the signature for multi-particle quantum interference is completely lost. 33.3% quantum efficiency for a detector is easily achievable (quantum efficiencies of higher than 90% has been reported). However, what makes α_e small is the temporal mode mismatch as a result of the Schwartz inequality (equation (54)). This will be the most important factor in the experimental investigation of this effect.

Next let us examine the possibility of using down-converted fields to produce a single-photon state $|\Psi\rangle$ as given in equation (37). It is known [27] that by gating the detection of the signal field upon the detection of a photon in the idler field, one can realize a single-photon state for a signal field. However, since the detection of the idler photon is irregular, it is hard to have an exact timing for matching the single-photon pulse to the temporal shape of the coherent field. In fact, for the stationary coherent field, one has to rely on the electronic response of the detector to achieve the temporal mode match as required for high equivalent quantum efficiency α_e [30]. On the other hand, with an ultra-fast pulse pumping the parametric down-conversion process, one may expect that everything will be synchronized by the ultra-fast pump pulse and the problem of temporal match will be solved. However, as we will see in the following, this is not always so because the down-converted fields are not transform limited. Only when we detect a much narrower band of the idler field to generate the gating current, will the gated signal field be a transform-limited single-photon field synchronized to the pumping pulse.

Now consider the two-photon state in equation (17) from the parametric down-conversion process. Suppose that detector Di registers an idler photon at time $t = t_i$. Then at the moment of detection, the state of the signal field collapses to a state that is obtained by projecting the two-photon state $|\Phi\rangle$ in equation (17) onto the state $|\psi(t_i)\rangle_i = K\hat{E}_i^{(-)}(t_i)|\mathrm{vac}\rangle_i$ with $\hat{E}_i^{(-)}(t_i) = \int \mathrm{d}\omega\, \hat{a}_i^\dagger(\omega) f_i(\omega) \mathrm{e}^{\mathrm{j}\omega t_i}/\sqrt{2\pi}$ where K is a normalization constant [31]:

$$
\begin{aligned}
|\Psi(t_i)\rangle_s &= {}_i\langle\psi(t_i)|\Phi\rangle \\
&= K' \int \mathrm{d}\Omega_1\, \mathrm{d}\Omega_2\, a_p(\Omega_1 + \Omega_2) \mathrm{e}^{\mathrm{j}(\Omega_1+\Omega_2)t_p} \\
&\quad \times \Phi(\Omega_1 + \Omega_2, (\Omega_1 - \Omega_2)/2) f_i(\Omega_2) \mathrm{e}^{-\mathrm{j}\Omega_2 t_i} \hat{a}_s^\dagger(\Omega_1)|\mathrm{vac}\rangle \\
&= \int \mathrm{d}\Omega_1\, \Psi(\Omega_1, t_i) \hat{a}_s^\dagger(\Omega_1)|\mathrm{vac}\rangle
\end{aligned}
\tag{55}
$$

with

$$
\Psi(\Omega_1, t_i) = K' \mathrm{e}^{\mathrm{j}\Omega_1 t_p} \int \mathrm{d}\Omega_2\, a_p(\Omega_1 + \Omega_2) \Phi(\Omega_1 + \Omega_2, (\Omega_1 - \Omega_2)/2) f_i(\Omega_2) \mathrm{e}^{-\mathrm{j}\Omega_2(t_i - t_p)} \tag{56}
$$

and $K' \equiv K^*/\sqrt{2\pi}$. Equation (55) has the form of equation (37), indicating that the gated field is in a transform-limited single-photon state. But the shape of the transform-limited pulse is determined by $\Psi(\Omega_1, t_i)$ which seems to depend on the detection time t_i. Let us consider the following two situations.

(i) If the bandwidth $\Delta\Omega_i$ of the filter f_i is much larger than $\Delta\Omega_p$, then we can make a change of variable $\Omega = \Omega_1 + \Omega_2$ in the integral in equation (56) and obtain

$$\Psi(\Omega_1, t_i) = K' e^{j\Omega_1 t_i} \int d\Omega\, a_p(\Omega)\Phi(\Omega, \Omega_1 - \Omega) f_i(\Omega - \Omega_1) e^{-j\Omega t_i}$$
$$\approx e^{j\Omega_1 t_i} K' a_p(0) f_i(-\Omega_1) \int d\Omega\, \Phi(\Omega, \Omega_1) e^{-j\Omega t_i} \qquad (57)$$

where we made the approximation by using the fact that the bandwidth $\Delta\omega_p$ of the pump field and the bandwidth $\Delta\Omega$ of the intrinsic down-conversion are much larger than $\Delta\Omega_p$ of $\Phi(\Omega_p, \Omega)$. Therefore, $\Psi(\Omega_1, t_i)$ has an extra phase of $e^{j\Omega_1 t_i}$, which indicates that the transform-limited pulse is peaked at t_i and has a pulse width determined by $\max(1/\Delta\Omega_i, 1/\Delta\Omega)$. Since the detection of the idler photon is uncertain to within $\Delta T_i \sim 1/\Delta\Omega_p \gg \max(1/\Delta\Omega_i, 1/\Delta\Omega)$, the location of the single-photon state is uncertain, which makes it impossible to match its shape to that of the coherent pulse.

(ii) On the other hand, when $\Delta\Omega_i$ is much smaller than $\Delta\Omega_p$ (which is also the situation for observing 100% visibility in fourth-order interference), we have from equation (56)

$$\Psi(\Omega_1, t_0) \approx K' a_p(\Omega_1) e^{j\Omega_1 t_p} \Phi(\Omega_1, \Omega_1/2) \int d\Omega_2\, f_i(\Omega_2) e^{-j\Omega_2(t_0 - t_p)}$$
$$= K' e^{j\Omega_1 t_p} a_p(\Omega_1)\Phi(\Omega_1, \Omega_1/2) F_i(t_i) \qquad (58)$$

with $F_i(\tau) \equiv \int d\Omega_2\, f_i(\Omega_2) e^{-j\Omega_2(\tau - t_p)}$. So $\Psi(\Omega_1, t_i)$ has both a bandwidth of $\Delta\Omega_p$ and a phase of $e^{j\Omega t_p}$ that are independent of the detection time t_i of the idler photon. Therefore, the transform-limited single-photon state is centred at t_p with a width of $1/\Delta\Omega_p$. The peak of the pulse is thus synchronized with the pump pulse. The key in obtaining a pump-synchronized transform-limited single-photon pulse is the fact that the bandwidth-limited detection of the idler photon lengthens the correlation time between the idler and signal photons, which ensures that the gated field has a bandwidth determined not by the down-conversion bandwidth, as the ungated signal field is, but by the equivalent pump bandwidth $\Delta\Omega_p$ (narrower than the down-converted bandwidth).

5. Summary and discussion

The process of parametric down-conversion with a nonstationary pumping field is analysed. For an ultra-short pumping field, because of the limitation by the phase-matching condition in the down-conversion process, only a small portion of the spectral components of the pumping field contributes to the generation of down-converted fields. The down-converted bandwidth takes the sum of the intrinsic bandwidth (corresponding to single-frequency pumping) and the equivalent pumping bandwidth (phase-matching bandwidth). However, the down-converted field has a temporal width determined by taking the reciprocal of the narrower one of the intrinsic bandwidth and the equivalent pumping bandwidth, thus making it impossible to form a transform-limited two-photon state.

A fourth-order interference experiment between the down-converted field and a coherent field is analysed. It is found that near 100% visibility is possible under the condition that optical filters are placed in front of the detectors with a filter bandwidth much narrower than both the intrinsic down-conversion bandwidth and the equivalent pumping bandwidth, otherwise a significant reduction of the visibility will occur. A multi-photon interference experiment involving a single-photon state and a strong coherent state is analysed. It is found that the temporal mode match between the transform-limited single-photon state and the coherent field is crucial for the observation of the interference effect. A possibility is investigated for utilizing the down-converted field to produce the single-photon state. It is found that when we gate the detection of the signal field on the detection of the idler field, a transform-limited single-photon state can be achieved, but with a centre position which is undetermined to within the detection time of the idler photon. However, such an uncertainty can be overcome by placing an optical filter in front of the idler detector with a filter bandwidth much narrower than both the intrinsic down-conversion bandwidth and the equivalent pumping bandwidth. The signal field generated in this way is in a transform-limited single-photon state synchronized with the pump pulse.

In both interference experiments involving the down-converted field and a coherent field, optical filters are required to be used in front of the detectors. The conditions for the observation of high visibility in interference are the same, that is, the filter bandwidth is much narrower than the intrinsic down-conversion bandwidth $\Delta\Omega$ and the equivalent pumping bandwidth $\Delta\Omega_p$ or $\Delta\Omega_f \ll \{\Delta\Omega, \Delta\Omega_p\}$. It is easy to satisfy $\Delta\Omega_f \ll \Delta\Omega$ since $\Delta\Omega$ is of the order of 10^{13} rad s^{-1}. However, to satisfy $\Delta\Omega_f \ll \Delta\Omega_p$, we need to look for those nonlinear media that have small group velocity dispersion. This is so because $\Delta\Omega_p$ is determined by $\pi/2l|(k'_p - k'_s)|$. Let us consider a BBO crystal, which has least dispersion among the available nonlinear crystals. From the index of refraction data [32], we find that $1/|(k'_p - k'_s)| \sim 25c$ at $\lambda = 0.8$ μm so that $\Delta\Omega_p \sim 3.8 \times 10^{12}$ rad s^{-1} for a crystal length of $l = 3$ mm. The best interference filter available so far has a bandwidth of $\Delta\Omega_f \sim 3 \times 10^{12}$ rad s^{-1}. This makes $\Delta\Omega_f \sim \Delta\Omega_p$. Although this does not prevent the observation of interference with finite visibility, it still requires $\Delta\Omega_f \ll \Delta\Omega_p$ to achieve a visibility close to 100%. A Fabry–Perot cavity can satisfy this, but it will substantially cut down the coincidence rate, approximately by a factor of $\Delta\Omega_f/\Delta\Omega_p$, and a Fabry–Perot cavity has many longitudinal modes, which makes it equivalent to a wide bandpass filter.

Under the ideal condition of $\alpha_e = 1$, the probability distribution in equation (53) has the same form as the absolute square of the wavefunction of the first excited state of a simple harmonic oscillator or equivalently the single-photon state. This indicates that by using optical tomography [33], we should be able to recover the Wigner function of a single-photon state [30] (which is a quasi-probability and completely describes a quantum system) thus making a complete measurement of the quantum state of a transform-limited pulse. For the nonideal case of $\alpha_e < 1$, the corresponding Wigner function has the form of

$$\begin{aligned} W(x, y) &= \frac{2}{\pi}\{(1-\alpha_e) + \alpha_e[2(x^2+y^2) - 1]\}\mathrm{e}^{-2(x^2+y^2)} \\ &= \frac{2}{\pi}[1 - 2\alpha_e + 2\alpha_e(x^2+y^2)]\mathrm{e}^{-2(x^2+y^2)}. \end{aligned} \tag{59}$$

Notice that at $x = y = 0$, the quasi-probability $W(0, 0) = 2(1 - 2\alpha_e)/\pi$ and becomes negative when $\alpha_e > 0.5$. The negativeness of the Wigner function is a manifestation of the quantum behaviour of a single-photon state.

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