Continuous-variable Bell-type correlations from two bright squeezed beams

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(Received 3 May 2001; revised manuscript received 22 Auguest 2001; published 11 December 2001)

In this paper we propose two surprisingly simple optical systems that display Bell-type correlations between the continuous variables of their subsystems. We first discuss how these two systems may be used to construct Bell-type correlations from two bright, amplitude-squeezed sources. Then the experimental viability of these systems is analyzed. Issues such as the conditions under which the systems display Bell correlations, sources of nontechnical as well as technical noise, signal-to-noise ratio and losses are discussed. Based on this discussion, we conclude that both of these systems are currently experimentally viable.

DOI: 10.1103/PhysRevA.65.012306 PACS number(s): 03.67.−a, 03.70.+k

I. INTRODUCTION

Applications of quantum information theory typically rely on entanglement, as characterized by nonclassical correlations between spatially separated subsystems of a quantum system [1,2]. Traditionally, discrete variables have been studied. However, fueled by fundamental issues as well as potential applications, attention has now also turned to the study of entanglement between the continuous variables of quantum systems [3–7].

Optics has turned out to be a fruitful test bed for quantum information demonstrations both for discrete variables [8–11] and continuous variables [12,13]. For the longer term quantum computation schemes in optics have now been identified in both variable domains [14,15].

Recently the link between the Bell correlations observed from down conversion sources through photon counting and the Einstein-Podolsky-Rosen (EPR) correlations observed from two-mode squeezed sources via homodyne detection has been identified [16]. In principle, this would allow Bell-type inequalities to be tested on bright squeezed beams [17]. More practically, these results open up new parameter regimes to experimental enquiry and possible applications. In particular, the pump conversion efficiencies possible in squeezed light production exceed by many orders of magnitude those possible with downconversion. On the other hand, the experiment proposed in Ref. [16] was complex, involving the locking of four independent squeezed sources.

The purpose of this paper is to propose and analyze two surprisingly simple experimental systems that display Bell-type correlations between the continuous variables of their subsystems. The central thrust of this paper is to analyze these systems in terms of their experimental viability. Hence we will discuss issues such as experimental simplicity, the degree of squeezing required, signal-to-noise ratios, and losses.

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Both of the systems proposed in this paper require only two bright squeezed sources with only moderate levels of amplitude squeezing. The first of these systems is one that has not previously been shown to exhibit Bell correlations. The second system proposed in this paper is essentially the squeezed source analog of the well-known Ou and Mandel [8] system proposed by Reid and Walls [18].

The paper is arranged in the following way. In Sec. II we define explicitly *Bell correlations* and review their measurement through continuous variable techniques. In Sec. III we introduce our new proposed schemes. In Sec. IV we discuss experimental issues and contrast our new schemes and the original proposal. We briefly discuss the extent to which the observation of Bell correlations constitutes a fundamental test of the validity of quantum mechanics in Sec. V and conclude in Sec. VI.

II. CONTINUOUS VARIABLE MEASURE OF BELL CORRELATIONS

Figure 1 shows a schematic diagram of a generic quantum optical Bell correlation experiment. Typically, Bell-type correlations are generated using a quantum optical system S that generates four-mode (two spatial and two polarization) correlated beams of light [19]. As indicated in Fig. 1, we shall designate the four modes of this system $\hat{A}_h$, $\hat{A}_v$, $\hat{B}_h$, and $\hat{B}_v$. A combination of polarizing optics, such as a half-wave plate and polarizing beamsplitter, can be used to decompose the two spatially distinct beams $\hat{A}$ and $\hat{B}$ into a polarization basis set $+,-$ at an angle θ to the original h,v basis set. This is given by the transformation

$$\hat{A}_+(\theta_A)=\cos\theta_A\hat{A}_h+\sin\theta_A\hat{A}_v,$$

$$\hat{A}_-(\theta_A)=\cos\theta_A\hat{A}_v-\sin\theta_A\hat{A}_h,$$

$$\hat{B}_+(\theta_B)=\cos\theta_B\hat{B}_h+\sin\theta_B B_v,$$

$$\hat{B}_-(\theta_B)=\cos\theta_B\hat{B}_v-\sin\theta_B\hat{B}_h. \qquad (1)$$

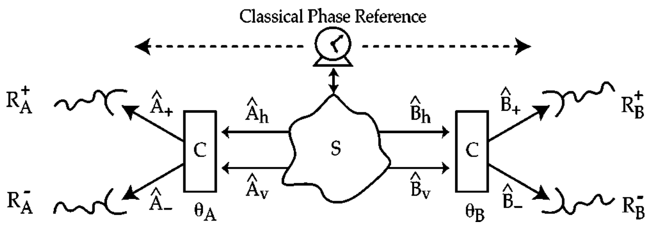

FIG. 1. A schematic diagram of a generic quantum optical system that can be used to test Bell's inequalities. The system S generates four-mode (two spatial and two polarization) correlated optical beams. The four modes of this system are $\hat{A}_h$, $\hat{A}_v$, $\hat{B}_h$, and $\hat{B}_v$. A combination of polarizing optics can be used to decompose the two spatially distinct beams $\hat{A}$ and $\hat{B}$ into a polarization basis set $+,-$ at an angle θ to the original h,v basis set. The results of these measurements are denoted $R^+(\theta)$ and $R^-(\theta)$, respectively.

Direct measurements of the photon number in the $+$ and $-$ paths of each beam can be made. As illustrated in Fig. 1, the results of these measurements are denoted $R^+(\theta)$ and $R^-(\theta)$, respectively. Central to the physics of this system is the issue that, for a general value of θ, it is not possible to determine from the measurements R^+ and R^- whether or not a particular photon took the path h or v. We can construct photon number correlations of the form

$$\langle R^{ij}(\theta_A\theta_B)\rangle = \langle R_A^i(\theta_A)R_B^j(\theta_B)\rangle = \langle \hat{A}_i^\dagger(\theta_A)\hat{A}_i(\theta_A)\hat{B}_j^\dagger(\theta_B)\hat{B}_j(\theta_B)\rangle, \tag{2}$$

where $i,j=+,-$. Any local realistic description of the photon number correlations R^{ij} would be bounded by the following Bell's inequality [20]:

$$B=|E(\theta_A,\theta_B)+E(\theta_A',\theta_B')+E(\theta_A',\theta_B)-E(\theta_A,\theta_B')|\leq 2, \tag{3}$$

where

$$E(\theta_A,\theta_B)=\frac{\langle R^{++}(\theta_A,\theta_B)\rangle+\langle R^{--}(\theta_A,\theta_B)\rangle-\langle R^{+-}(\theta_A,\theta_B)\rangle-\langle R^{-+}(\theta_A,\theta_B)\rangle}{\langle R^{++}(\theta_A,\theta_B)\rangle+\langle R^{--}(\theta_A,\theta_B)\rangle+\langle R^{+-}(\theta_A,\theta_B)\rangle+\langle R^{-+}(\theta_A,\theta_B)\rangle}. \tag{4}$$

We define our subsystems as exhibiting *Bell correlations* if their correlation functions, R^{ij}, satisfy $B>2$.

The correlation functions are defined above in terms of photon number measurements such as $\hat{A}_i^\dagger(\theta_A)\hat{A}_i(\theta_A)$. However, in Ref. [16] it was shown that these correlations could be decomposed into a series of quadrature amplitude measurements on the subsystems and their measurement environment by using the equivalence

$$\hat{A}_i^\dagger\hat{A}_i \equiv 4(\hat{A}_i^\dagger\hat{A}_i-\hat{V}_i^\dagger\hat{V}_i)=(\hat{X}^i_{A;1})^2+(\hat{X}^i_{A;2})^2-(\hat{X}^i_{V;1})^2-(\hat{X}^i_{V;2})^2, \tag{5}$$

where $\hat{X}_{F;1}=\hat{F}+\hat{F}^\dagger$ represents the amplitude quadrature operator, $\hat{X}_{F;2}=\imath(\hat{F}-\hat{F}^\dagger)$ represents the phase quadrature operator, and $\hat{V}_i$ is a vacuum mode such that $\langle\hat{V}_i^\dagger\hat{V}_i\rangle=0$. Experimentally, $\hat{X}_{F;1}$ and $\hat{X}_{F;2}$ can be measured using the balanced homodyne technique [19]. Substituting Eq. (5) into Eq. (2) and assuming our fields have Gaussian statistics allows R^{ij} to be reduced to the following sum of second-order quadrature amplitude correlation functions,

$$R^{ij}=\frac{1}{16}[2(\langle\hat{X}^i_{A;1}\hat{X}^j_{B;1}\rangle^2+\langle\hat{X}^i_{A;2}\hat{X}^j_{B;2}\rangle^2+\langle\hat{X}^i_{A;2}\hat{X}^j_{B;1}\rangle^2+\langle\hat{X}^i_{A;1}\hat{X}^j_{B;2}\rangle^2)+V_{A;1}V_{B;1}+V_{A;2}V_{B;2}+V_{A;2}V_{B;1}+V_{A;1}V_{B;2}-2V_v(V_{A;1}+V_{A;2})-2V_v(V_{B;1}+V_{B;2})+4V_v^2], \tag{6}$$

where $V_{F;k}=\langle(\hat{X}_{F;k})^2\rangle$ for $k=1,2$.

Equation (6) is essentially a polarization-dependent correlation function. High polarization fringe visibility is required if the system is to exhibit Bell correlations. The first four terms of Eq. (6) are the signal terms that depend on the angles of the polarizers θ_A and θ_B. The next four terms represent polarization-independent noise. The final three terms are purely quantum mechanical in origin. Ideally the quantum mechanical terms cancel the polarization-independent ones, thus producing high polarization-signal visibility.

From a quantum information point of view the violation of Eq. (3) indicates the presence of strong entanglement between the subsystems. The presence of such entanglement is viewed as a resource for various quantum information tasks. The main aim of this paper is to examine what experimental systems and requirements are needed to observe Bell correlations in the continuous variable quantum information context. A brief discussion of the significance of continuous variable Bell correlations in fundamental tests of quantum mechanics is made in Sec. V.

It was shown in Ref. [16] that continuous-variable measurements of the system reproduced in Fig. 2 would demonstrate Bell correlations. We denote this system as S_1.

III. BELL CORRELATIONS FROM TWO BRIGHT SQUEEZED BEAMS

Figure 3 shows a schematic diagram of the first of the Bell systems proposed in this paper. Two amplitude

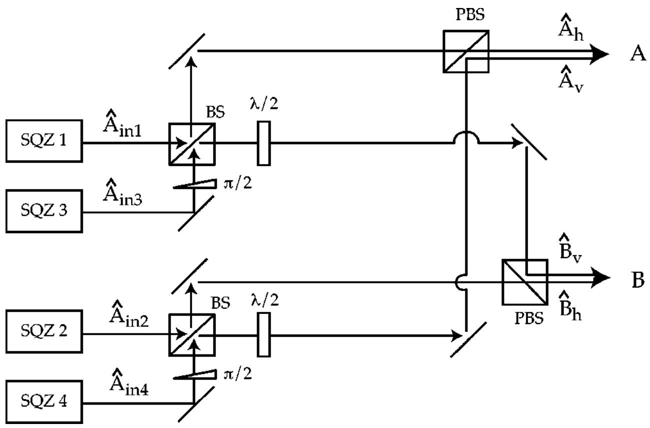

FIG. 2. A reproduction of the system proposed in Ralph *et al.* Two horizontally polarized, bright, amplitude-squeezed sources $\hat{A}_{in1}$ and $\hat{A}_{in3}$ are incident on a 50% transmitting beam splitter with a 90° phase shift relative to each other. The two fields emerging from this beam splitter display EPR correlations. The two input fields $\hat{A}_{in2}$ and $\hat{A}_{in4}$ are combined in a similar fashion. The polarizations of the two beams transmitted through the two beam splitters are rotated by 90°. The horizontally polarized field from the upper beam splitter and the vertically polarized field from the lower beam splitter are recombined on a polarizing beam splitter to form the two output modes $\hat{A}_h$ and $\hat{A}_v$. The other two fields are recombined in a similar fashion to form the two output modes $\hat{B}_h$ and $\hat{B}_v$. This system is designated S_1 in the text.

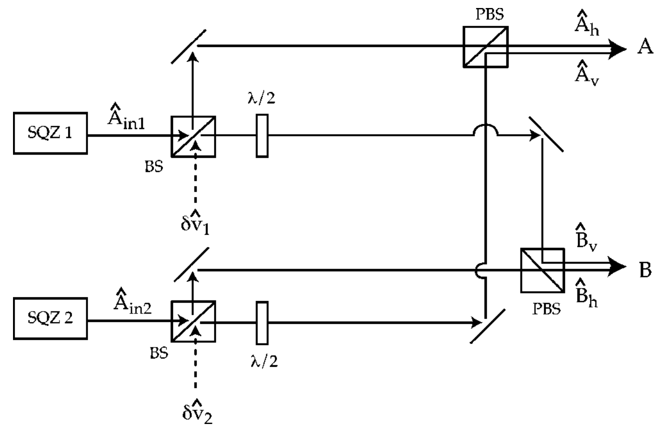

FIG. 3. A schematic diagram of our first proposal. Two horizontally polarized, bright, amplitude-squeezed sources $\hat{A}_{in1}$ and $\hat{A}_{in2}$ are incident on two separate 50% transmitting beam splitters. Two vacuum fields, $\hat{A}_{v1}$ and $\hat{A}_{v2}$ are introduced at these two beam splitters. The outputs of these beam splitters are manipulated and recombined in the same fashion as the outputs of the two beam splitters in Fig. 2 to form the four correlated output modes $\hat{A}_h$, $\hat{A}_v$, $\hat{B}_h$, and $\hat{B}_v$. This system is designated S_2 in the text.

squeezed, horizontally polarized, bright beams are incident on two 50% transmitting beam splitters. Two of the four resultant beams have their polarizations rotated by 90 °. The beams are then recombined as indicated in Fig. 3 on polarizing beam splitters such that two spatially distinct beams are generated each with two polarization modes. We denote this to be the system S_2.

For bright squeezed systems, it is generally more convenient to work in the Fourier domain. We decompose an arbitrary operator $\hat{F}=\overline{F}+\delta\hat{F}$ and then work with the frequency-dependent quadrature amplitude and phase fluctuation operators, $\delta X_{F;1}(\omega)=\delta F+\delta F^{\dagger}$ and $\delta X_{F;2}(\omega)=\imath(\delta F-\delta F^{\dagger})$, respectively. The absence of circumflexes implies the transformation into the Fourier domain. The remainder of this paper will be couched in terms of correlations between the fluctuations of the fields. Using the asymmetric beam splitter phase convention, the quadrature fluctuation operators for the four modes of the system S_2 are

$$\delta X_{A_h;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in1};k}-\delta X_{v1;k}),$$

$$\delta X_{A_v;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in2};k}+\delta X_{v2;k}),$$

$$\delta X_{B_h;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in2};k}-\delta X_{v2;k}),$$

$$\delta X_{B_v;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in1};k}+\delta X_{v1;k}), \tag{7}$$

where $k=1,2$ and the input fields are defined in Fig. 3.

The correlation functions for the system shown in Fig. 3 are

$$R_{S_2}^{++}(\omega)=R_{S_2}^{--}(\omega)=\frac{1}{16}\left[\frac{(V_{in;1}-V_{v;1})^2+(V_{in;2}-V_{v;2})^2}{2}(\cos\theta_A\sin\theta_B+\sin\theta_A\cos\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\right],$$

$$R_{S_2}^{+-}(\omega)=R_{S_2}^{-+}(\omega)=\frac{1}{16}\left[\frac{(V_{in;1}-V_{v;1})^2+(V_{in;2}-V_{v;2})^2}{2}(\cos\theta_A\cos\theta_B-\sin\theta_A\sin\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\right], \tag{8}$$

where, in Fourier space, the variances are given by $V_{F;k}=\langle|\delta X_{F;k}|^2\rangle$ for $k=1,2$. Also, we have set the quadrature amplitude and phase variances of the two input squeezers to be the same and given by $V_{in;1}$ and $V_{in;2}$, respectively. The quadrature amplitude and phase variances of the vacuum inputs are denoted by $V_{v;k}$ for $k=1,2$ and, for our definitions of the quadrature amplitudes, are equal to unity.

The polarization visibility is maximized when we assume

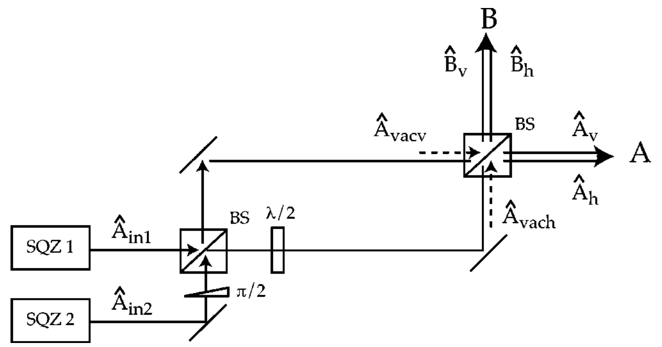

FIG. 4. A schematic diagram of our second proposal. Two horizontally polarized, bright, amplitude-squeezed sources $\hat{A}_{in1}$ and $\hat{A}_{in2}$ are incident on a 50% transmitting beam splitter with a 90° phase shift relative to each other. The two fields emerging from this beam splitter display EPR correlations. The polarization of the field transmitted through the beam splitter is rotated by 90°. The two fields recombined on a polarization-independent 50% transmitting beam splitter to form the four correlated output modes $\hat{A}_h$, $\hat{A}_v$, $\hat{B}_h$, and $\hat{B}_v$. This system is designated S_3 in the text.

that the two input fields are weakly squeezed minimum uncertainty states. Under these conditions, we find that

$$E(\theta_A,\theta_B)\approx -\cos 2(\theta_A+\theta_B). \tag{9}$$

Choosing the angles $\theta_A=3\pi/8$, $\theta_A'=\pi/8$, $\theta_B=-\pi/4$, and $\theta_B'=0$, we find that $B=2\sqrt{2}$ thereby violating Eq. (3). Comparison of Fig. 3 to Fig. 2 highlights the elegant simplicity of the system proposed in this paper. System S_2 as proposed here requires only two amplitude-squeezed sources. Compare this to the four coherently related, amplitude-squeezed sources and phase locking loops required in S_1.

There is another way of making use of two bright squeezed sources to generate Bell correlations. Figure 4 shows schematically our second proposed two-squeezer Bell system. This system is essentially the squeezed source analog of the well-known Ou and Mandel [8] Bell experiment proposed by Reid and Walls [18]. We denote this system as S_3. The quadrature fluctuations of the fields in this system are

$$\delta X_{A_h;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in1};k}-\delta X_{A_{in2};k}-\delta X_{A_{vach};k}),$$

$$\delta X_{A_v;k}=\frac{1}{\sqrt{2}}(-\delta X_{A_{in2};k}-\delta X_{A_{in2};k}+\delta X_{A_{vacv};k}),$$

$$\delta X_{B_h;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in1};k}-\delta X_{A_{in2};k}+\delta X_{A_{vach};k}),$$

$$\delta X_{B_v;k}=\frac{1}{\sqrt{2}}(\delta X_{A_{in1};k}+\delta X_{A_{in2};k}+\delta X_{A_{vacv};k}), \tag{10}$$

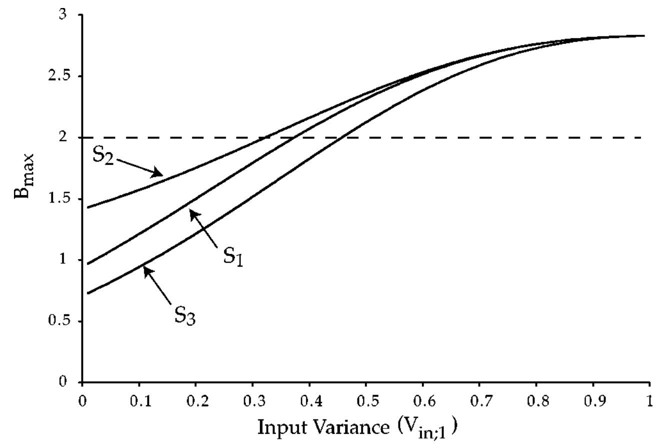

FIG. 5. A plot of B_{max} as a function of the input squeezing for the three systems S_1, S_2, and S_3.

where $k=1,2$ and the input fields are defined in Fig. 4. The correlation functions for system S_3 are

$$\begin{aligned}R_{S_3}^{++}(\omega)=R_{S_3}^{--}(\omega)=\frac{1}{16}\bigg[&\frac{(V_{in;1}-V_{in;2})^2}{4}(\cos\theta_A\sin\theta_B\\&-\sin\theta_A\cos\theta_B)^2\\&+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}(\cos\theta_A\cos\theta_B\\&-\sin\theta_A\sin\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\bigg],\\R_{S_3}^{+-}(\omega)=R_{S_3}^{-+}(\omega)=\frac{1}{16}\bigg[&\frac{(V_{in;1}-V_{in;2})^2}{4}(\cos\theta_A\cos\theta_B\\&+\sin\theta_A\sin\theta_B)^2\\&+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}(\cos\theta_A\sin\theta_B\\&+\sin\theta_A\cos\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\bigg],\end{aligned} \tag{11}$$

where we have again set the quadrature amplitude and phase variances of the two input squeezers to be the same and given by $V_{in;1}$ and $V_{in;2}$, respectively. For the angles $\theta_A=3\pi/8$, $\theta_A'=\pi/8$, $\theta_B=\pi/4$, and $\theta_B'=0$, this system will also violate Eq. (3).

IV. A DISCUSSION OF SOME EXPERIMENTAL ISSUES

A continuous variable system will exhibit Bell correlations provided that two key criteria are satisfied. First, the outputs of the Bell system must be such that the polarization-independent terms in Eq. (6) cancel. That satisfied, the correlation between the outputs of the Bell system must be nonzero. Because Eq. (4) is normalized, any degree of correlation will suffice. In broad terms, the systems discussed in this paper satisfy both of these criteria. In experimental implementations of either of the schemes proposed in Fig. 3 or

Fig. 4, there remain some issues to be discussed.

Let us begin by giving the equivalent to Eqs. (8) and (11) for S_1,

$$R^{++}_{S_1}(\omega)=R^{--}_{S_1}(\omega)=\frac{1}{16}[(V_{in;1}-V_{in;2})^2(\cos\theta_A\sin\theta_B+\sin\theta_A\cos\theta_B)^2+(V_{in;1}+V_{in;2}-2)^2],$$

$$R^{+-}_{S_1}(\omega)=R^{-+}_{S_1}(\omega)=\frac{1}{16}[(V_{in;1}-V_{in;2})^2(\cos\theta_A\cos\theta_B-\sin\theta_A\sin\theta_B)^2+(V_{in;1}+V_{in;2}-2)^2], \quad (12)$$

where this equation for S_1 was generated from the starting point that the quadrature fluctuations of the fields in S_1 are given by Eqs. (7) with $\delta X_{v1,2;1}$ replaced by $\delta X_{in3,4;2}$ and $\delta X_{v1,2;2}$ replaced by $\delta X_{in3,4;1}$.

The polarization-independent terms in Eqs. (8), (11), and (12) can be made to be approximately zero. However, the degree of cancellation depends strongly on the degree of input squeezing. Figure 5, which plots the maximum value of B as a function of the input squeezing for systems S_1, S_2, and S_3, shows that strongly squeezed input fields destroy the violation of Eq. (3). The physical interpretation of a weakly squeezed field is that correlated photons are generated in pairs. As a field becomes more and more strongly squeezed, significant numbers of photons are generated in groups of four, six, and so on. These higher-order groupings destroy the correlations that lead to a violation of Eq. (3). Hence weak squeezing is required to generate Bell correlations.

As Fig. 5 indicates, each of the Bell systems is affected differently by strongly squeezed inputs. The equations used to generate Fig. 5 are

$$B_{max,S_1}=2\sqrt{2}\left|\frac{(V_{in;1}-V_{in;2})^2}{(V_{in;1}-V_{in;2})^2+2[V_{in;1}+V_{in;2}-2]^2}\right|,$$

$$B_{max,S_2}=2\sqrt{2}\left|\frac{(V_{in;1}-1)^2+(V_{in;2}-1)^2}{(V_{in;1}-1)^2+(V_{in;2}-1)^2+[V_{in;1}+V_{in;2}-2]^2}\right|,$$

$$B_{max,S_3}=2\sqrt{2}\left|\frac{(V_{in;1}-V_{in;2})^2}{(V_{in;1}-V_{in;2})^2+3[V_{in;1}+V_{in;2}-2]^2}\right|, \quad (13)$$

where the terms arising from incomplete cancellation of noise are illustrated in Eqs. (13) through the use of square brackets and the correlation between the outputs is illustrated through the use of parentheses. Equations (13) reveal that the tradeoff between the magnitude of the correlations and the cancellation of noise is different for each system. This is due to the very different ways in which the Bell correlations are generated in each of these systems. We find that system S_2 as proposed in this paper can demonstrate Bell correlations for $0.32<V_{in;1}<1$. In contrast, the original continuous variable system S_1 exhibits Bell correlations for $0.37<V_{in;1}<1$ and S_3 as shown in Fig. 4 will only allow $0.46<V_{in;1}<1$.

Unfortunately, weak squeezing comes at the cost of the magnitude of the correlation signal. This is because strong correlation requires strong squeezing. The correlation fringes as described by Eq. (6) are normalized to the product of the quantum noise limits (QNL) for each of the measurements $R^{+}(\theta)$ and $R^{-}(\theta)$. Thus, one way of quantifying the signal size is to find the magnitude of the maximum of the correlation fringes described by Eq. (6). We will denote this quantity F. For the three systems discussed in this paper, we find that

$$F_{S_1}(\omega)=\frac{(V^2_{in;1}-1)^2}{16V^2_{in;1}},$$

$$F_{S_2}(\omega)=F_{S_3}(\omega)=\frac{(V_{in;1}-1)^2(V^2_{in;1}+1)}{32V^2_{in;1}}, \quad (14)$$

where we have assumed minimum uncertainty input states. Equations (14) are plotted in Fig. 6.

As expected, Fig. 6 reveals that the signal for all of the systems increases as the squeezing is increased. This figure also shows that the signal for system S_1 is approximately four times that for systems S_2 and S_3 over the squeezing range of interest. The signal size in systems S_2 and S_3 should indeed be one quarter that of S_1 at very low levels of squeezing. This is most simply illustrated by examining the normalized correlation coefficient C between two outputs of the systems. For example, C_{S_1}, C_{S_2}, and C_{S_3} between the two output modes A_h and B_v for each of the systems are [19,21,22]

$$C_{S_1}=\frac{|\langle\delta X_{A_h;1}\delta X_{B_v;1}\rangle|}{\sqrt{V_{A_h;1}V_{B_v;1}}}=\frac{(V^2_{in;1}-1)}{(V^2_{in;1}+1)},$$

$$C_{S_2}=C_{S_3}=\frac{(V_{in;1}-1)}{(V_{in;1}+1)}, \quad (15)$$

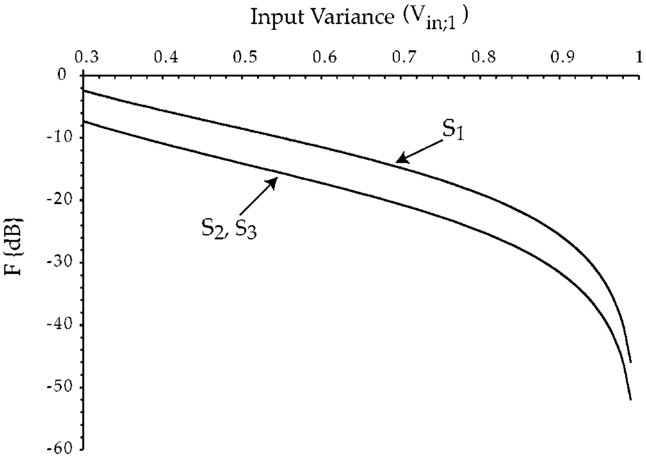

FIG. 6. A plot of F, the maximum of the correlation fringe as a function of the input squeezing for the three systems S_1, S_2, and S_3. F is a measure of the magnitude of the correlation signal compared to the quantum-noise limit at both detectors.

where we again assume minimum uncertainty states. Typically, $|C|=1$ is taken to be an indicator of complete correlation between two fields. At the weak levels of squeezing required to violate Eq. (3), the normalized correlation coefficient for the fields in system S_1 is twice that of the fields in systems S_2 and S_3. The correlation fringes are proportional to C^2 and therefore $F_{S_1}\approx 4F_{S_2}=4F_{S_3}$ is to be expected.

The reduced signal size for S_2 compared to S_1 is a direct result of the vacuum fluctuations introduced at the beam splitters in Fig. 3. It is well known that vacuum fluctuations reduce the correlation between the outputs of a beam splitter. A cursory inspection of Fig. 3 reveals that vacuum fluctuations are introduced into system S_2 at the input beam splitters. Also, in S_3, vacuum fluctuations for each polarization mode are introduced at the second beam splitter. Indeed, the second squeezing input used at the input beam splitters of Fig. 2 is used precisely to increase the correlation by a factor of 2 and hence increase F by a factor of 4.

Out of interest, we can also compare the quantity $F(\omega)$ to the total number of photons $P(\omega)$ at a certain Fourier frequency. Quite generally, the number of photons in a field at a Fourier frequency is given by $P(\omega)=(V_{A;1}+V_{A;2}-2)/4$. We find that, for moderate levels of squeezing, every photon at a certain Fourier frequency leads to a coincidence for S_1 and that one in every two photons leads to a coincidence for systems S_2 and S_3.

The question still remains, are the correlation signals generated by each of these systems experimentally detectable? The issue of detectability is essentially one of determining whether or not the correlation fringes can be resolved in the presence of the polarization-independent noise. The ability to resolve two different levels in a power spectrum is proportional to the resolution bandwidth of the measurement [22,23]. Thus, in principle, arbitrarily small correlation signals can be detected provided that an arbitrarily narrow resolution bandwidth, or equivalently, an arbitrarily long integration time, is employed.

Turning to some technical issues, let us consider noise introduced in the detection process. We can model the effect of detector noise by rewriting the quadrature fluctuation operators $\delta X^i_{P;k}(\omega)$ as detected quadrature fluctuation operators

$$\delta X^i_{P;k,det}=\delta X^i_{P;k}+\delta X^i_{det,P}, \tag{16}$$

where $P=A,B$, $i=+,-$, and $k=1,2$. We assume that the electronic noise $\delta X^i_{det,P}$ added at each detector is uncorrelated with respect to any other noise source.

The effect of electronic noise is to raise the noise floor of the correlation measurement. For S_1, the polarization-independent noise terms in Eqs. (12) would become $(V_{in;1}+V_{in;2}-2+2V_{det})^2$. For S_2 and S_3, the polarization independent noise terms in Eqs. (8) and (11), respectively would become $(V_{in;1}+V_{in;2}-2+4V_{det})^2/4$. In these equations, $V_{det}(\omega)$ is the power of the electronic noise at a certain frequency ω normalized to the QNL for the measurement. System S_1 is less susceptible to electronic noise because the nontechnical noise floor of the measurement is greater than that for systems S_2 and S_3.

The increased noise floor of the correlation function will cause the polarization fringe visibility to be reduced. A reduction in the polarization fringe visibility will have two consequences for these measurements. First, as the polarization fringe visibility is reduced, the resolution bandwidth required to resolve the correlation fringes will increase. Second, as we have already seen, the quantity B_{max} depends very heavily on the polarization fringe visibility. Electronic noise will make the measured B_{max} less than the ideal B_{max} as given in Eqs. (13).

In the absence of electronic noise, the noise floor of the measurement is minimized for $V_{in;1}\approx 1$. Consequently, the presence of electronic noise will be most noticeable as the squeezing is reduced. The effect of electronic noise at $V_{in;1}\approx 1$ is exacerbated by the fact that the correlation signal is also very small at low levels of squeezing. Figures 7(a)–7(c) show plots of B_{max} for each of the systems as a function of $V_{in;1}$ for three different values of electronic noise as compared to the shot noise at each detector. Figure 7 illustrates that each of the systems has an optimum value of $V_{in;1}$ at which it should be operated.

Another important experimental issue is the presence of technical noise on the inputs to the systems. In much of the discussion thus far it has been assumed that the inputs to each of the Bell systems were minimum uncertainty states. However, this assumption is not necessarily true in an experiment. In all of the systems discussed in this paper the polarization-independent noise floor of the experiment is at a minimum for minimum uncertainty states. Consequently, if the input is not a minimum uncertainty state the polarization fringe visibility of the correlation function will be decreased. This will again affect the values of B_{max} and the required resolution bandwidth for the measurement. The effect of not having minimum uncertainty states is quite strong and could prove to be a significant technical issue.

One final issue that must always be considered in experiments is loss. As an example, consider the situation where

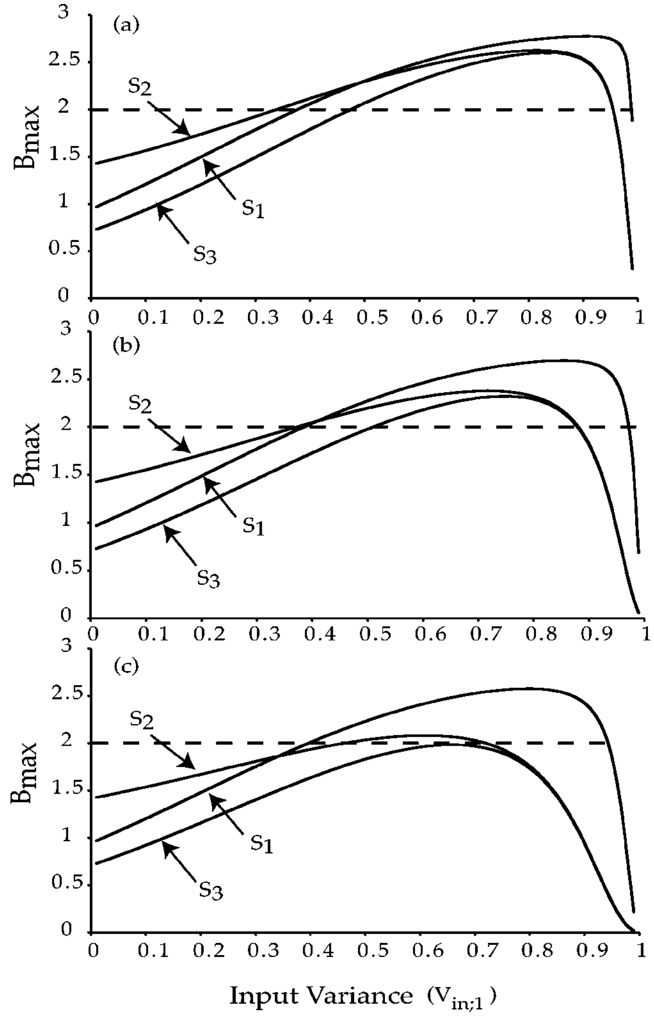

FIG. 7. A plot of B_{max} as a function of the input squeezing for the three systems S_1, S_2, and S_3. In (a) it is assumed that electronic noise is 20 dB below the QNL, in (b) it is assumed that electronic noise is 16 dB below the QNL, and in (c) it is assumed that electronic noise is 13 dB below the QNL.

loss occurs equally in all four arms of system S_2. The correlation functions in the absence of detector noise for S_2 would become

$$R^{++}=R^{--}=\frac{\epsilon^2}{16}\left[\frac{(V_{in;1}-V_{v;1})^2+(V_{in;2}-V_{v;2})^2}{2}(\cos\theta_A\sin\theta_B+\sin\theta_A\cos\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\right],$$

$$R^{+-}=R^{-+}=\frac{\epsilon^2}{16}\left[\frac{(V_{in;1}-V_{v;1})^2+(V_{in;2}-V_{v;2})^2}{2}(\cos\theta_A\cos\theta_B+\sin\theta_A\sin\theta_B)^2+\frac{(V_{in;1}+V_{in;2}-2)^2}{4}\right],$$

where ϵ represents the power transmission of the equivalent beam splitter that is used to model loss. A similar result holds for all of the systems discussed in this paper.

Thus we find that losses will reduce the correlation between the system outputs. The effect of losses on the noise floor of the experiment is a little less straightforward. First, losses will make both $V_{in;1}$ and $V_{in;2}$ appear closer to the QNL. This will reduce the noise floor of the correlation measurement. However, at the same time, losses will make the input state apparently less like a minimum uncertainty state. This will cause the noise floor of the correlation measurement to increase slightly. Interestingly, the net effect is that, in the absence of electronic noise, the noise floor of the correlation measurement will be reduced at the same rate as the correlation signal.

For all of the Bell systems discussed in this paper, we find that the normalized function given in Eq. (4) is unchanged compared to the lossless case in the absence of detector noise. Hence, these systems can still demonstrate Bell correlations. However, the signal size F and the nontechnical noise floor are both reduced by the factor ϵ^2 compared to the lossless case. Thus not only will the required resolution bandwidth be smaller than the lossless case, the effect of electronic noise will be more significant.

Experimental implementation of the two systems proposed in this paper would require two bright, amplitude-squeezed sources. Typically, this technology is very mature and source instability would be a relatively minor source of experimental noise. Hence, in the measurements proposed in this paper, we expect that the major technical concern would be the electronic noise of the photodetection system. Based on the plots shown in Fig. 7, we require that electronic noise be at least 13 dB less than shot noise for S_2 and ideally 16 dB below shot noise. For S_3, the electronic noise must be at least 16 dB below shot noise. Although stringent, such a requirement is currently technically feasible (see, for example, Ref. [24]).

By contrast, the system shown in Fig. 2 requires four bright, amplitude-squeezed sources and numerous phase-locking loops. In this case, source stability is a critical issue and it is unlikely that it would have an insignificant effect on the inputs to the system. Thus, although this system is less susceptible to detector noise, it would be much less likely that the inputs to the system would be minimum uncertainty states. Given the technical challenges posed by S_1 and the high sensitivity to source stability, initial experimental demonstrations of continuous variable Bell correlations are probably more likely using either of the two-squeezer systems proposed in this paper.

V. SIGNIFICANCE FOR FUNDAMENTAL TESTS

The original Bell inequality was formulated as a fundamental test of quantum mechanics, specifically designed to observationally delineate between quantum mechanics and all local hidden variable theories. From this point of view the Bell inequality of Eq. (3) only applies given various requirements on the subsystems and their measurement such as space-like separation, high-efficiency detection, etc. Typically all requirements are not met for a particular experimental manifestation and certain, hopefully reasonable, assumptions need to be made. For example, in the recent ion-trap experiments [25] it was not possible to achieve spacelike separations. Thus, in order to interpret the results as a fundamental test, it had to be assumed that the observed Bell correlations were not established by the passage of some unknown, hidden but causal, signals. We briefly consider these types of issues here.

In order to interpret the observation of Bell correlations obtained from the quadrature correlations of Eq. (6) as a fundamental test two further requirements and one assumption are needed over and above those normally required in a photon counting experiment [26,27].

Requirement 1: The addition of the local oscillator in a correctly balanced homodyne arrangement does not add extraneous noise to the measurement of a signal beam. This can and has been confirmed experimentally to an accuracy of over 6 dB below the vacuum noise limit. This prevents correlated noise being added via the local oscillators or through some conspiracy between the detectors. This can be viewed as a technical requirement of the experimental apparatus.

Assumption: The ensemble averages constructed from the homodyne measurements $R_A^i(\theta_A)=(\hat{X}_{A;1}^i)^2+(\hat{X}_{A;2}^i)^2-(\hat{X}_{V;1}^i)^2-(\hat{X}_{V;2}^i)^2$ and its correlations with a similarly defined $R_B^i(\theta_B)$ are identical to those which would be obtained by measuring directly $R_A^i(\theta_A)'=4(\hat{A}_i^\dagger\hat{A}_i-\hat{V}_i^\dagger\hat{V}_i)$ and a similarly defined $R_B^i(\theta_B)'$ in a photon counting experiment. In other words, we assume that Eq. (5) is true for any plausible local hidden variable theory. The validity of the Bell inequality relies on the positivity of the individual measurements making up the ensemble averages. Because $\hat{X}_{A;1}^i$ and $\hat{X}_{A;2}^i$, etc., do not commute, the individual measurement results cannot be determined from the quadrature measurements. Thus we must rely on Eq. (5) to draw conclusions about the properties of the individuals. This assumption is true for classical optics, semiclassical optics, stochastic electrodynamics and quantum optics. It is difficult to see how a consistent theory could be constructed that did not agree with this assumption.

Requirement 2: The mode V_i is measured to have zero intensity ($\langle V_i^\dagger V_i\rangle=0$). This requirement ensures that in a counting experiment, $R_A^i(\theta_A)'$, etc., would be positive. Hence our ensemble averages are derived from positive individuals and the Bell inequality is valid. The fact that the vacuum is a zero-temperature bath at optical frequencies is well established and has been assumed throughout this paper. However, for a fundamental test, a real-time intensity measurement of the vacuum would be required to avoid conspiracy loop holes.

Assertion: For all local hidden variable theories satisfying our assumption either Requirement 1 will fail or; Requirement 2 will fail or; the Bell inequality of Eq. (3) cannot be violated. For example classical optical theory obeys our assumption and satisfies both requirements, but cannot violate Eq. (3). While stochastic electrodynamics satisfies the assumption, Requirement 1 and can produce Bell correlations, but does not satisfy Requirement 2.

VI. CONCLUSION

Drawing on recent theoretical advances in the treatment of continuous variable systems [16], we have proposed two systems that can be used to observe Bell-type correlations between the continuous variables of their subsystems. We briefly discussed conditions under which these Bell correlations constitute a fundamental test of quantum mechanics. However, the primary aim of this paper was to investigate the experimental condititions under which Bell correlations could be observed.

In the first system proposed in this paper (denoted S_2 in the text), two bright, amplitude-squeezed inputs are split into two to generate two sets of partially correlated beams. After polarization manipulation and recombination, the outputs of this system are shown to display Bell correlations. This system is similar to that originally proposed for continuous variable observations of Bell correlations [16].

The original system (denoted S_1 in the text) made use of four squeezed input fields to generate two pairs of EPR correlated states that, after polarization manipulation and recombination, could then be used to demonstrate Bell correlations. Interestingly, the correlations generated in the two-squeezer system proposed in this paper are significantly smaller than the original system. In spite of this, the nonclassical correlations between the system outputs are sufficient to demonstrate Bell correlations.

In the second system proposed in this paper (denoted S_3), two bright, amplitude-squeezed inputs are combined to generate a single pair of EPR correlated beams. The polarization of one of these beams is rotated and they are then recombined on a polarization-independent beam splitter. This system is essentially the bright squeezed source analog of the photon counting experiment of Ou and Mandel [8].

Having discussed how the correlations may be constructed, the focus of this paper then shifted to an analysis of the performance of these systems. To provide a reference point, both of the systems proposed in this paper, S_2 and S_3, were compared to S_1, the system originally shown to display Bell correlations between the continuous variables of its subsystems [16].

First, it was shown that excessive squeezing introduces noise that reduces the polarization signal visibility of the correlations. This noise is nontechnical in nature and arises from the fundamental properties of strongly squeezed fields. It was shown that S_2 displayed Bell correlations over the widest range of squeezing values followed by S_1 and then S_3. Consequently, the first system proposed in this paper was shown to be the most robust against sources of nontechnical noise.

Unfortunately, even in the absence of technical noise, moderate or low squeezing comes at the cost that the correlations between the system outputs is relatively small. Defining the signal magnitude to be the maximum of the correlation fringes, we found that the original system S_1 would have four times the signal magnitude of the systems proposed in this paper, S_2 and S_3, at very low levels of squeezing.

Finally, potential sources of technical noise such as detector noise, the stability of the squeezed sources, and losses were discussed. It was shown that there is an optimum point at which to operate an experiment that will result in the best tradeoff between large correlations and a good signal-to-noise ratio. For the two systems proposed in this paper, the use of two bright, amplitude-squeezed sources rather than four would mean that systems S_2 and S_3 would be much less subject to technical problems associated with source stability. It was also shown that, in the absence of electronic noise, all three systems would be similarly affected by losses. It was concluded that the Bell correlations of systems S_2 and S_3 would be detectable using currently available experimental apparatus.

ACKNOWLEDGMENTS

T.C.R. acknowledges useful discussions with P. Grangier and W. Munro. This work was supported in part by the Australian Research Council.

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