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Schroedinger cat states and optimum universal quantum cloning by entangled parametric amplification

Francesco De Martini [*], Valentina Mussi, Fabio Bovino

Dipartimento di Fisica and Istituto Nazionale di Fisica della Materia, Università 'La Sapienza', Roma 00185, Italy

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Abstract

The new process of *quantum-injection* by a single-photon in a pure quantum superposition state into an optical parametric amplifier operating in *entangled* configuration is adopted to generate an all optical multiphoton *Schroedinger-cat* state which is largely detectable against the squeezed-vacuum noise. The invariance properties of the OPA interaction hamiltonian show that, under certain conditions, the device may act as a universal quantum cloning machine (UQCM) of the input qubits. Preliminary results here reported show the first experimental realization of such a device based on stimulated emission process in an optical parametric amplifier. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

In the last decade the process of optical parametric interaction in a nonlinear crystal has been at the core of all experiments testing the nonlocal character of quantum mechanics or taking advantage of quantum nonlocality to manipulate the quantum information [1]. This is for instance the case of the tests of violation of Bell inequalities [2], of the nonlocality proofs without inequalities [3], of quantum state-teleportation [4] and of all processes generally belonging to the chapter of *nonlocal entangled interferometry* [5,6]. However, in spite of the well known properties of the method and of the wide generality of its potential applications, in all these experiments it has only been adopted as a spontaneous parametric-converter (SPDC) i.e., to generate correlated photon couples or 'squeezed vacuum' fields [1]. Only recently one of us has proposed a new approach to the problem based on the amplifying / squeezing operation of the travelling wave optical parametric amplifier (OPA) operating in a novel 'entangled' configuration and, most important, initiated by a new quantum dynamical interaction process here referred to as *quantum-injection*, i.e. provided by an input field whose *P-Representation* does not exist as a tempered solution [7]. For instance the character of *quantum-injection* may be provided by the subpoissonian character of a single photon in the Fock state $n = 1$ in a quantum superposition of polarization, or momentum, states. Sometimes we shall refer to this single particle superposition state as the input *qubit*. This photon may belong to a couple generated by an additional SPDC process e.g. in a *Φ-phase tunable* entangled state of linear polarization $\boldsymbol{\pi}$, defined in a Hilbert space of dimensionality 2×2. The key idea

[*] Corresponding author. E-mail: demartini@axcasp.caspur.it

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of the present work relates to the possibility of somewhat forcing a large number of emitted photons to share this quite interesting superposition phenomenology, i.e., to create a Schroedinger cat multiphoton state [9]. In the language of modern quantum mechanics this procedure consists of the general process of '*cloning*' a large number M of qubits starting from one or a limited number $N < M$ of input qubits, by taking advantage of the *unitary* character of the transformation accounting for the parametric amplification process [8]. As said, this quantum amplification process is expected to give rise to a multiparticle *entangled Schroedinger-Cat (S-Cat)* condition. It is well known that the generation of classically distinguishable quantum states is a major endeavor of modern physics that has long been the object of extensive theoretical investigation and of few recent experimental studies mostly in the field of atomic physics [10–13].

The present paper is organized as follows. A detailed account of the quantum dynamics of the *entangled quantum-injected OPA* and the analysis of the output field, indeed a multiparticle *S-Cat* field, is reported in Section 2. Then we explore in the Section 3 the conditions under which our system leads to the first physical implementation of a universal quantum cloning machine (UQCM). We also show that this machine has indeed been realized experimentally by our system. In Section 4 we report an *exact*, closed form evaluation of the Wigner functions of the output S-Cat fields emphasizing the formal role taken by the single-photon quantum injection scheme with respect to the superposition properties of the S-Cat, a system which is *decoherence-free* in the ideal case. A theory of the *first-order* and *second- order* optical correlation functions of the parametrically generated field, also given in Section 4, suggests a straightforward interferometric method for a direct single-beam detection of the S-Cat quantum superposition process and a new class of multi-particle Bell-inequality experiments [14].

2. Quantum injection in the non-degenerate optical parametric amplifier

Consider the diagram shown in Fig. 1. Two equal and equally oriented nonlinear (NL) crystals are excited by two beams derived from a common laser beam with wavelength (wl) λ_p. Crystal 1 is the SPDC source of linear polarization-*entangled* (or: $\boldsymbol{\pi}$-*entangled)* photon couples emitted, with wl $\lambda = 2\lambda_p$ over the modes $i = 1,3$ (i.e. $\boldsymbol{k}_1, \boldsymbol{k}_3$) determined by two fixed pinholes. Assume that the SPDC source creates one couple and that one photon of it is injected into crystal 2 which provides the parametric amplification. The second correlated photon, detected over the mode 3, provides the gate pulse of the overall *conditional* experiment. Let us analyze the dynamics of the system.

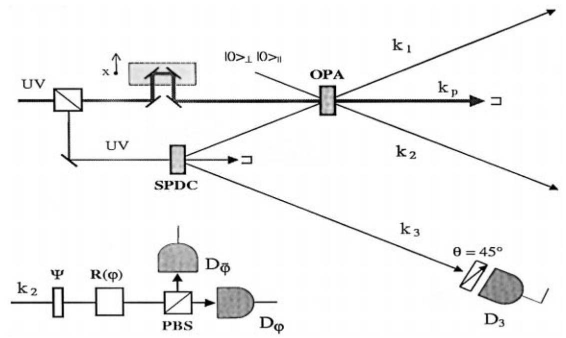

Fig. 1. Optical configuration of the quantum-injected, entangled optical parametric amplifier realizing the process of multiphoton quantum superposition.

For a Type II NL crystal operating in noncollinear configuration the overall amplification process taking place over $\boldsymbol{k}_j$ is contributed by two equal and independent amplifiers OPA_A and OPA_B inducing unitary transformations respectively on two couples of time dependent field operators: $\hat{a}_1(t) \equiv \hat{a}(t)_{1\perp}$, $\hat{a}_2(t) \equiv \hat{a}(t)_{2\parallel}$ and $\hat{b}_1(t) \equiv \hat{a}(t)_{1\parallel}$, $\hat{b}_2(t) \equiv \hat{a}(t)_{2\perp}$ for which, at the initial time of the interaction and for any i, $j = 1,2$, is: $[\hat{a}_i, \hat{a}_j^\dagger] = [\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$ and $[\hat{a}_i, \hat{b}_j^\dagger] = 0$, being: $\hat{a}_i \equiv \hat{a}_i(0)$, $\hat{b}_i \equiv \hat{b}_i(0)$ the field operators at $t = 0$. In the following we shall refer to this new type of optical parametric amplifier as an *entangled* OPA. The Hamiltonian of the overall interaction may be expressed in the general form: $H_I = i\hbar g[A^\dagger - e^{i\Psi} B^\dagger] + \text{h.c.}$ where: $A^\dagger \equiv \hat{a}_1(t)^\dagger \hat{a}_2(t)^\dagger$, $B^\dagger \equiv \hat{b}_1(t)^\dagger \hat{b}_2(t)^\dagger$, $g \equiv \chi t$ is a real number expressing the *amplification gain*, and χ the coupling term proportional to the product of the 2nd-order NL susceptibility of the crystal and of the *pump* field, here assumed classical and undepleted by the parametric interaction. The interaction time, it may be determined in our case by the length L of the NL crystal. The quantum dynamics of OPA_A and OPA_B is expressed by the mutually commuting, unitary *squeezing operators*: $U_A(t) = \exp[g(A^\dagger - \hat{A})]$ and $U_B(t) = \exp[-g e^{i\Psi}(B^\dagger - \hat{B})]$ implying the following Bogoliubov transformations for the field operators evaluated at time t:

$$\begin{bmatrix} \hat{a}_1(t) \\ \hat{a}_2(t)^\dagger \end{bmatrix} = \begin{bmatrix} C & S \\ S & C \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2{}^\dagger \end{bmatrix};$$

$$\begin{bmatrix} \hat{b}_1(t) \\ \hat{b}_2(t)^\dagger \end{bmatrix} = \begin{bmatrix} C & \tilde{S} \\ \tilde{S}^* & C \end{bmatrix} \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2{}^\dagger \end{bmatrix}, \tag{1}$$

where: $C \equiv \cosh g$, $S \equiv \sinh g$, $\tilde{S} \equiv -e^{i\Psi} S$ and $\hat{a}_i \equiv \hat{a}_i(0)$, $\hat{b}_i \equiv \hat{b}_i(0)$ are evaluated at the initial interaction time, $t = 0$. Note that the Hamiltonian H_I can be expressed as: $H_I = i\hbar g(\sigma_+^A + \sigma_+^B) +$ h.c. in terms of the spin operators: $\sigma_+^A \equiv \hat{A}^\dagger$, $\sigma_-^A \equiv -\hat{A}$, $\sigma_+^B \equiv -e^{i\Psi} \hat{B}^\dagger$, $\sigma_-^B \equiv e^{-i\Psi}\hat{B}$ mutually connected by the standard commutors: $[\sigma_+^A, \sigma_-^A] = \sigma_z^A$, $[\sigma_+^B, \sigma_-^B] = \sigma_z^B$, $[\sigma_\pm^A, \sigma_z^A] = \mp 2\sigma_\pm^A$, $[\sigma_\pm^B, \sigma_z^B] = \mp 2\sigma_\pm^B$ which lead to the definitions: $\sigma_z^A = 1 + \hat{a}_1{}^\dagger \hat{a}_1 + \hat{a}_2{}^\dagger \hat{a}_2$, $\sigma_z^B = 1 + \hat{b}_1{}^\dagger \hat{b}_1 + \hat{b}_2{}^\dagger \hat{b}_2$. A further important physical property of H_I, not directly connected to the former spin representation, consists of its rotational invariance under general SU(2) transformations for $\Psi = 0$. In this case is easy to prove, in virtue of the Schwinger's model and of the Baker–Hausdorff lemma, that H_I is invariant under general rotations of the polarization vectors $\boldsymbol{\pi}$ of the input photons associated with the modes $i = 1,2$ [15]. In other words, the unitary SU(2) transformation R expressing the $\boldsymbol{\pi}$ rotations over the i-modes,

$$\begin{bmatrix} \hat{a}_1'^\dagger \\ \hat{b}_1'^\dagger \end{bmatrix} = R^\dagger \begin{bmatrix} \hat{a}_1^\dagger \\ \hat{b}_1^\dagger \end{bmatrix} R = \begin{bmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{bmatrix} \begin{bmatrix} \hat{a}_1^\dagger \\ \hat{b}_1^\dagger \end{bmatrix};$$

$$\begin{bmatrix} \hat{b}_2'^\dagger \\ \hat{a}_2'^\dagger \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{bmatrix} \begin{bmatrix} \hat{b}_2^\dagger \\ \hat{a}_2^\dagger \end{bmatrix}, \tag{2}$$

with $|\alpha|^2 + |\beta|^2$, does not affect the form of H_I but merely induces a change of the field components with the corresponding *rotated* (i.e. primed) ones, i.e. $H_I = i\hbar g[\hat{A}^\dagger - \hat{B}^\dagger] +$ h.c. $\Rightarrow H_I = i\hbar g[\hat{A}'^\dagger - \hat{B}'^\dagger] +$ h.c. where: $\hat{A}'^\dagger \equiv \hat{a}_1'{}^\dagger \hat{a}_2'{}^\dagger$, $\hat{B}'^\dagger \equiv \hat{b}_1'{}^\dagger \hat{b}_2'{}^\dagger$. The angular momenta implied by the R transformation over the i-modes are: $J_{1y} = i\frac{1}{2}(J_{1-} - J_{1+}) = i\frac{1}{2}\hbar(\hat{b}_1^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{b}_1)$; $J_{2y} = i\frac{1}{2}\hbar(\hat{a}_2^\dagger \hat{b}_2 - \hat{b}_2^\dagger \hat{a}_2)$, $J_{1z} = (2\hbar)^{-1}[J_{1+}, J_{1-}] = \frac{1}{2}\hbar(\hat{a}_1{}^\dagger \hat{a}_1 - \hat{b}_1{}^\dagger \hat{b}_1)$, $J_{2z} = \frac{1}{2}\hbar(\hat{b}_2{}^\dagger \hat{b}_2 - \hat{a}_2{}^\dagger \hat{a}_2)$. Note that H_I is also invariant under any phase shift affecting the vertical field components on the two i-modes (i.e., $\hat{a}_1$ and $\hat{b}_2$) respect the corresponding horizontal ones ($\hat{b}_1$ and $\hat{a}_2$). This leads to the conclusion that for $\Psi = 0$ the *entangled* OPA squeezer/amplifier works equally well for *any* single photon input state, i.e. represented by any point lying on the corresponding Poincaré'sphere. In short, the device is a *universal* squeezer/amplifier.

In order to appreciate the role of the *quantum injection* process within the OPA dynamics, we find convenient at this stage to give more details about the setup shown in Fig. 1, the one corresponding to the actual experiment presently being carried out in our Laboratory. The two equal and equally oriented NL crystals are 1 mm thick beta-barium-borate (BBO) slabs, cut for Type II phase matching, excited by two beams derived from a common UV laser beam at a wavelength (wl) $\lambda_p = 2\pi|\boldsymbol{k}_p|^{-1} =$ 400 nm. The SPDC quantum-injector is provided by a Type II $\Phi-$*phasetunable* generator of linear polarization ($\boldsymbol{\pi}$)-entangled photon couples. The detection system consists of a birefringent plate Ψ, a $\boldsymbol{\pi}$-*rotator* $R(\varphi)$, a polarizing-beam-splitter PBS and two cooled photomultipliers. In the experiment a similar system is inserted on mode 2. We found that the entanglement phase $|\Phi|$ of the output state of the couple can be easily tuned over the range $0 - \pi$ by rotating by an angle ψ the crystal around the excitation axis $\boldsymbol{k}_p$, $\Phi(\psi)$ being a linear function [6]. In order to prevent any EPR type state reduction that may affect the overall superposition process and then destroy the S-cat at the outset, the photon emitted over the output mode 3 is filtered by a polarization analyzer with axis oriented at 45° to the horizontal (t.h.) before being detected by D_3 [14]. An alternative solution for quantum injection, successfully tested in the experiment, is provided by a Type I NL crystal 1 feeding the OPA by a single photon with $\boldsymbol{\pi}$ oriented at 45°, the other photon exciting D_3 without any $\boldsymbol{\pi}$-*selection*. In both cases, a click at D_3 opens a gate selecting all registered outcomes, thus providing the *conditional* character of the overall experiment, as said. The photon emitted by the crystal 1 provides the quantum-injection into the OPA, physically consisting of the other crystal. The input state to our amplifier system may be expressed in terms of superposition of Fock states associated with the j-modes ($j=1,2$) and with the two $\boldsymbol{\pi}$ components respectively parallel and orthogonal (t.h.):

$$|\Psi_1\rangle = 2^{-1/2}|0\rangle_{2\perp}\otimes|0\rangle_{2||}\otimes\left[|1\rangle_{1\perp}\otimes|0\rangle_{1||}+e^{i\Phi}|0\rangle_{1\perp}\otimes|1\rangle_{1||}\right]. \quad (3)$$

The output state is found by use of the overall evolution operator $U_{AB}=U_A(t)U_B(t)$ and of the following *disentangling* theorem, to be expressed in terms of the formerly defined spin operators $\sigma_\pm^A$, $\sigma_\pm^B$ [16]: $\exp k(\sigma_+ + \sigma_-) = \exp(\sigma_\pm \tanh k)\times(\cosh k)^{\mp\sigma_z}\exp(\sigma_\mp \tanh k)$. The output state may be generally cast in the form:

$$|\Psi\rangle_{out} \equiv G\{|\Psi_B(0)\rangle\otimes|\Psi_A(1)\rangle + e^{i\Phi}|\Psi_A(0)\rangle\otimes|\Psi_B(1)\rangle\}, \quad (4)$$

where: $G\equiv(\sqrt{2}C^3)^{-1}$; $|\Psi_B(0)\rangle \equiv \sum_{n=0}^{\infty}\times(-1)^n e^{in\Psi}\Gamma^n|n\rangle_{1||}\otimes|n\rangle_{2\perp}$, $|\Psi_A(0)\rangle \equiv \sum_{n=0}^{\infty}\Gamma^n|n\rangle_{1\perp}\otimes|n\rangle_{2||}$, $\Gamma\equiv S/C$. Note that $(\Gamma^{2n}/C^2)\equiv P_n=(\bar{n})^n/[1+\bar{n}]^{(1+n)}$ is a thermal distribution accounting for the squeezed-vacuum noise generated independently by $OPA_A$and OPA_B with equal average photon numbers: $\bar{n}=\bar{n}_A=\bar{n}_B=|S^2|$ [1]. The two states expressed in (4) as: $|\Psi_A(1)\rangle = \sum_{m=0}^{\infty}\Gamma^m\sqrt{m+1}|m+1\rangle_{1\perp}\otimes|m\rangle_{2||}$, $|\Psi_B(1)\rangle = \sum_{m=0}^{\infty}(-1)^m e^{im\Psi}\Gamma^m\sqrt{m+1}|m+1\rangle_{1||}\otimes|m\rangle_{2\perp}$ represent the effect of the single-photon quantum-injection, $N=1$. Since this sum is extended over the complete set of n-states the appeal to the *macroscopic* quantum coherence is justified. We should add here, for reference, an estimate of the actual value of the parameter Γ attainable in typical experiments. In the actual experiment described later in Section 4 involving a 150 femtosecond, mode-locked TI:Sa laser with an average power = 0.3 W and a 1 mm BBO NL crystal is found: $\Gamma=0.017$. However, by a previous amplification of the TI:Sa laser by a Coherent REGA9000 regenerative amplifier, now in operation in our laboratory, the value $\Gamma\approx0.35$ is obtained. This value can be further increased by a tighter focusing in the crystal. The output state function, written in the form $|\Psi\rangle=[|\underline{\Psi_A}\rangle+e^{+i\Phi}|\underline{\Psi_B}\rangle]$ with $|\underline{\Psi_A}\rangle\equiv|\Psi_B(0)\rangle\otimes|\Psi_A(1)\rangle$ and $|\underline{\Psi_B}\rangle\equiv|\Psi_A(0)\rangle\otimes|\Psi_B(1)\rangle$ expresses the condition of quantum superposition between two *pure*, multi-particle states originating, through unitary OPA transformations, from the input single-particle state $|\Psi_1\rangle$, keeping in this process its original phase Φ. In facts, any unitary transformation may generally transform but not cancel the relevant quantum properties of the input state, such as superposition and entanglement, even in presence of a *noisy* process of particle amplification as in our case. Furthermore, most important, since the output state $|\Psi\rangle_{out}$ is not factorizable in terms of linear polarization $\boldsymbol{\pi}$-*states*, it is a $\boldsymbol{\pi}$-*entangled* state. As such it is expected to reproduce in the multi-particle regime the striking quantum nonseparability and Bell-type nonlocality properties of the microscopic (i.e. 2-particle) systems [14,17]. The $\boldsymbol{\pi}$-*entanglement* properties of the output state can be investigated experimentally either by a multi-particle Bell inequality experiment or more directly by the ad hoc optical configuration already successfully adopted for the case of a $\boldsymbol{\pi}$-*entanglement* of a single photon couple [5,6].

3. Optimal universal cloning by entangled parametric amplification

In the language of modern quantum measurement theory, the *amplification* of single particle states (*qubits)* quoted explicitly at the outset of our first investigation on the squeezing-amplification of the *entangled OPA* [7], precisely consists of the process of *cloning* a quantum input state. After the first formulation of the *no cloning theorem* in 1982 by Wootters and Zurek [18], which deals with the *exact* cloning of a state, only very recently the solution of problem of the *optimum cloning* has been brought to a satisfactory degree of completion [8]. As it is well known, the problem consists of the establishment of the physical conditions that *optimize* the figure of merit of the cloning process, i.e. maximize its *Fidelity* $F\equiv\langle\Psi|\rho_{out}|\Psi\rangle$ where $|\Psi\rangle$ is the input state and ρ_{out} is the density matrix of the output field [19]. It is found that the *entangled amplifier / squeezer* device shown in Fig. 1 indeed represents today the only physical solution which can implement the opti-

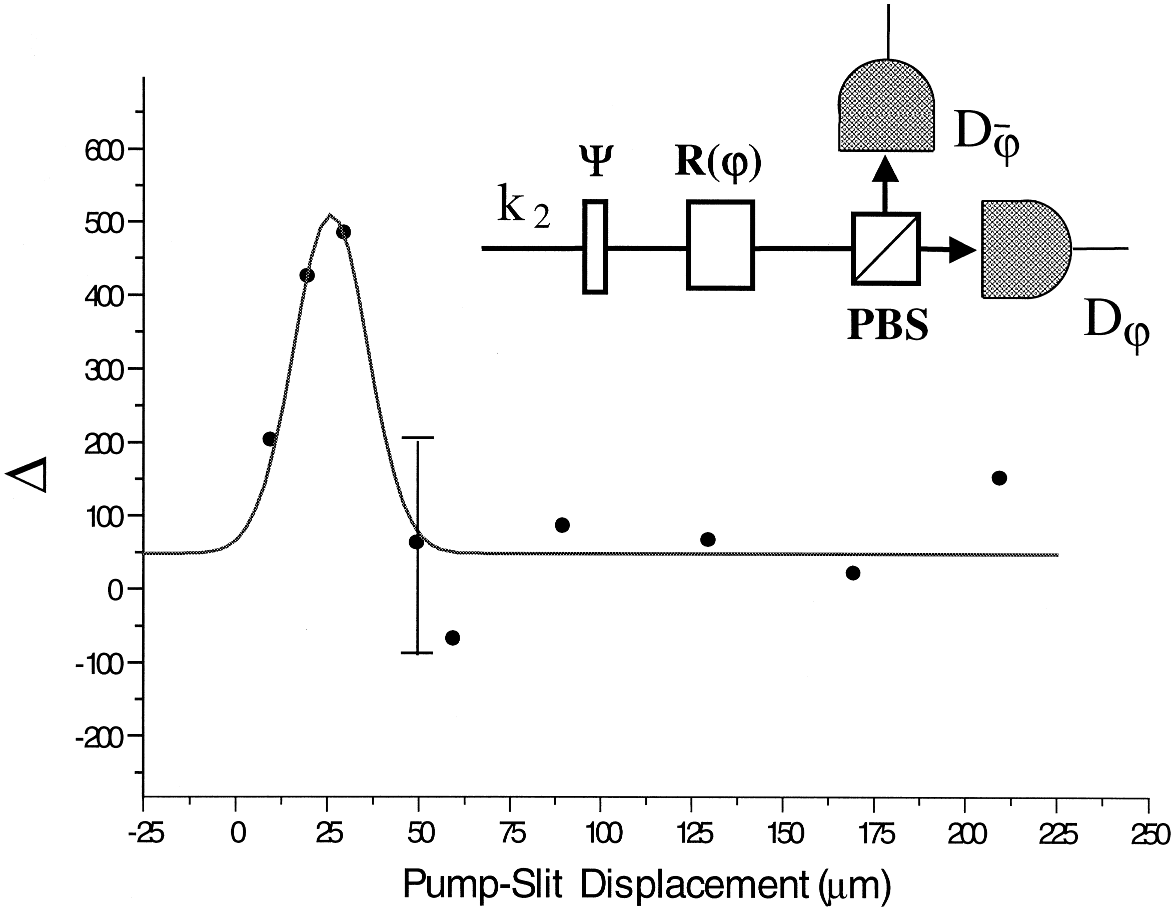

Fig. 2. Experimental realization of the cloning of $M=2$ photons under *quantum injection* of $N=1$ photons in an entangled OPA. The quantity Δ is the difference between the results of the measurement of the 1st-order correlation functions $G_2^{(1)}(\varphi)$ by the detectors $D_{2\varphi}$ and $D_{2\bar{\varphi}}$ within a conditional experiment aimed at the determination of the fringe pattern related to the S-Cat quantum state superpostition.

mum universal quantum cloning machine (UQCM) [20]. In the early paper [7] we pointed out that the process of amplification of the vacuum fluctuations associated with the input modes was to be considered as the unavoidable source of output 'squeezed vacuum' noise that determines the essential quantum limit to all possible *figures of merit* of the device e.g., the signal-to-noise ratio of any amplification or squeezing process or, more generally, the *Fidelity* F of any unitary transformation affecting the input qubits. Let's evaluate F for the process of cloning a fixed number N of input qubits into a fixed number $M>N$. Taking advantage of the *universal* character of the amplification/ squeezing process for $\Psi=0$ considered in Section 2, this may be done by *quantum injecting* the Fock N-state $|N\rangle$ into *any* input mode of the *entangled* OPA, e.g. into $\boldsymbol{k}_1$ with a vertical $\boldsymbol{\pi}$. In the latter case the overall input state is: $|N\rangle_{1\perp}=[\hat{a}_{1\perp}^{\dagger}(0)]^N(N!)^{-1/2}|\Psi_0\rangle$, where: $|\Psi_0\rangle=\{|0\rangle_{1\perp}|0\rangle_{1||}\otimes|0\rangle_{2\perp}|0\rangle_{2||}\}$. Since $U_{AB}\hat{a}_{1\perp}^{\dagger}(0)U_{AB}^{\dagger}=\hat{a}_{1\perp}^{\dagger}(t)$, the output state corresponding to a N photon input on mode $(1\perp)$ may be expressed as: $|\Psi_N\rangle=[\hat{a}_{1\perp}^{\dagger}(t)]^N(N!)^{-\frac{1}{2}}|\Psi_0\rangle$. This state may be evaluated explicitly by the disentangling techniques we adopted in Section 2. We can further elect an output state $|\Psi_N\rangle$ corresponding to the emission of a *fixed* number of M photons, i.e. *clones*, over the output mode $\boldsymbol{k}_1$with *both* polarizations $\{\perp, ||\}$. This state may be expressed as:

$$|\Psi_N\rangle_M = C^{-2}\Gamma^{M-N}\sum_{m:0}^{M-N}(-1)^m e^{im\Psi}\sqrt{\binom{M-m}{N}} \times|M-m\rangle_{1\perp}|m\rangle_{1||}|m\rangle_{2\perp}|M-N-m\rangle_{2||}. \quad (5)$$

If we set $C=1$ and $\Psi=0$, this state results to be identical to the one obtainable by the optimum cloning transformations considered by Buzek, Hillery and Werner [19,20]. We may then attribute to our case the value of the *Fidelity* evaluated by these authors: $F=(NM+M+N)(MN+2M)^{-1}$. Note that for $N=1$ is found: $F=(2M+1)/(3M)=(2/3)+1/(3M)$. This shows that for zero cloning (i.e. $g=0$, $M=N$): $F=1$; for *minimum* cloning (i.e. $M=N+1$): $F=(N+1)(N+2)^{-1}+N[(N+1)(N+2)]^{-1}$; for very large cloning (i.e. $M\gg N$): $F=(N+1)(N+2)^{-1}$. Note that for $N=1$, the value of the Fidelity decreases from $F=5/6$ for $M=N+1$, to the asymptotic value $F=4/6$ for $M\gg N$. This decrease of F is of course ascribable to the multiple entanglement established between an increasing number of clones. In Fig. 2 is reported the first experimental realization of the process of *quantum injection* of $N=1$ photons in the state given by Eq. (3)) into an OPA amplifier. The reported signal corresponds to an estimated cloning of $M=2$ particles by *entangled* parametric stimulated emission. The OPA process outlined in the present Section may of course represent an interesting demonstration of the modern theory of *cloning* of quantum states. However its practical use in the domain of modern quantum information, where the cloning theory is today being developed, is highly questionable. The point here is that N photons, in spite of being often properly associated with spin-$\frac{1}{2}$ particles, are actually *indistinguishable* objects when belonging to the same *mode*. Consequently they are represented by a Bose *symmetrized* Fock state, whose information content is: $N+1$ bits. This is a far smaller number than the value 2^N bits corresponding to the case of N *distinguishable* spins-$\frac{1}{2}$ that can be addressed and read separately without ambiguity.

4. Wigner and field's correlation functions

In order to inspect at a deeper lever the above results, consider the Wigner function of the output field for the configuration shown in Fig. 1 and assume in the expression of H_I the phase $\Psi=\pm\pi$. Evaluate first the symmetrically-ordered characteristic function of the set of complex variables $(\eta_j,\eta_j^*,\xi_j,\xi_j^*)\equiv\{\eta,\xi\}$, $(j=1,2)$:

$$\chi_s\{\eta,\xi\}\equiv\langle\Psi_1|D[\eta_1(t)]D[\eta_2(t)] \times D[\xi_1(t)]D[\xi_2(t)]|\Psi_1\rangle$$

expressed in terms of the *displacement* operators: $D[\eta_j(t)]\equiv\exp[\eta_j(t)\hat{a}_j(0)^\dagger-\eta_j^*(t)\hat{a}_j(0)]$, $D[\xi_j(t)]\equiv\exp[\xi_j(t)\hat{b}_j(0)^\dagger-\xi_j^*(t)\hat{b}_j(0)]$ where: $\eta_1(t)\equiv(\eta_1 C-\eta_2^* S)$; $\eta_2(t)\equiv(\eta_2 C-\eta_1^* S)$; $\xi_1(t)\equiv(\xi_1 C-\xi_2^* S)$; $\xi_2(t)\equiv(\xi_2 C-\xi_1^* S)$. The Wigner function, expressed in terms of the corresponding complex phase-space variables $(\alpha_j,\ \alpha_j^*,\ \beta_j,\ \beta_j^*)\equiv\{\alpha,\beta\}$ is the eight-dimensional Fourier transform of $\chi_s\{\eta,\xi\}$, namely:

$$W\{\alpha,\beta\} = \pi^{-8}\int\!\!\int\!\!\int\!\!\int d^2\eta_1 d^2\eta_2 d^2\xi_1 d^2\xi_2\ \chi_s\{\eta,\xi\} \times\exp\left\{\sum_j\left[\eta_j^*\alpha_j-\eta_j\alpha_j^*+\xi_j^*\beta_j-\xi_j\beta_j^*\right]\right\}, \quad (6)$$

where $d^2\eta_j\equiv d\eta_j d\eta_j^*$, etc. By a lengthy application of operator algebra and integral calculus we could evaluate analytically in closed form either $\chi_s\{\eta,\xi\}$ and $W\{\alpha,\beta\}$. The final *exact* expression of the Wigner function may be cast in the form:

$$W\{\alpha,\beta\} = -\overline{W_A}\{\alpha\}\overline{W_B}\{\beta\}\left[1-\left|e^{i\Phi}\Delta_A\{\alpha\}+\Delta_B\{\beta\}\right|^2\right], \quad (7)$$

where $\Delta_A\{\alpha\}\equiv 2^{-1/2}(\gamma_{A+}-i\gamma_{A-})$, $\Delta_B\{\beta\}\equiv 2^{-1/2}(\gamma_{B+}-i\gamma_{B-})$ are expressed in terms of the squeezed variables: $\gamma_{A+}\equiv(\alpha_1+\alpha_2^*)e^{-g}$; $\gamma_{A-}\equiv i(\alpha_1-\alpha_2^*)e^{+g}$; $\gamma_{B+}\equiv(\beta_1+\beta_2^*)e^{-g}$; $\gamma_{B-}\equiv i(\beta_1-\beta_2^*)e^{+g}$. The Wigner functions $\overline{W_A}\{\alpha\}\equiv 4\pi^{-2}\exp\left(-\left[|\gamma_{A+}|^2+|\gamma_{A-}|^2\right]\right)$; $\overline{W_B}\{\beta\}\equiv 4\pi^{-2}\exp\left(-\left[|\gamma_{B+}|^2+|\gamma_{B-}|^2\right]\right)$ definite positive over the

4-dimensional spaces $\{\alpha\}$ and $\{\beta\}$ represent the effect of squeezed-vacuum, i.e. emitted respectively by OPA_A and OPA_B in absence of any injection. Inspection of Eq. (6) shows that precisely the quantum superposition character of the injected state $|\Psi_0\rangle$ determines the dynamical quantum superposition of the devices OPA_A and OPA_B, the ones that otherwise act as *uncoupled* and *independent* objects. From another perspective, since the quasi-probability functions $\overline{W_A}\{\alpha\}$, $\overline{W_B}\{\beta\}$ corresponding to the two macrostates $|\Psi_A\rangle$ and $|\Psi_B\rangle$ in absence of quantum superposition are defined in two totally separated and independent spaces, their respective 'distance' in the overall phase-space of the system $\{\alpha,\beta\}$ can be thought of as 'macroscopic', as generally required by any standard S-cat dynamics in a 2-dimensional phase-space [10]. The link between the spaces $\{\alpha\}$ and $\{\beta\}$ is provided by the quantum superposition term in Eq. (6) $2\mathrm{Re}[e^{i\Phi}\Delta_A\{\alpha\}\Delta_B^*\{\beta\}]$. This term provides precisely the first-order quantum interference of the macrostates $|\Psi_A\rangle$ and $|\Psi_B\rangle$. In addition, and most important, Eq. (6) shows the non definite positivity of $W\{\alpha,\beta\}$ over its definition space. This assures the overall quantum character of our multi-particle, quantum-injected amplification scheme [1,21]. We may recognize that these last properties of the overall Wigner function of our system do coincide with the standard formal requirements of any Schroedinger-cat apparatus [21]. In this connection, let us state the following:

(a) The ability of the system to create a first-order interference fringe pattern is a necessary but *not sufficient* condition for any genuine S-cat behavior.

(b) The two interfering macrostates, identified by two corresponding gaussian-like peaks of the Wigner function must be clearly distinguishable, i.e., the phase space '*distance*' between the peaks must be larger than their average width.

Note that this condition implies necessarily the system's ability to produce a first-order interference pattern while the inverse argument is not necessarily true, as said.

The striking quantum mechanical features of the system are also revealed by the 1st and 2nd-order correlation functions of the OPA output fields. Refer to the inset of Fig. 1. Before detection over the modes $\boldsymbol{k}_j (j=1,2)$ the fields are phase-shifted by $\Psi_j = (\psi_{j\perp} - \psi_{j||})$ by birefringent plates and filtered by $\boldsymbol{\pi}$-analyzers with axes oriented at the angles: $45° + \varphi_j$ (t.h.). Each $\boldsymbol{\pi}$-analyzer may consist of the combination of a Fresnel-rhomb $\boldsymbol{\pi}$-rotator, $R(\varphi)$ and of a polarizing beam splitter. The field associated with the mode $\boldsymbol{k}_j$ is detected at the space-time positions x_j by two *linear* detectors $D_{j\varphi}$ and $D_{j\overline{\varphi}}$ with $\overline{\varphi} \equiv \varphi + 90°$.

The 1st-order correlation-functions $G_j^{(1)}(x_j,x_j) \equiv \langle \Psi_0 | N_j(t) | \Psi_0 \rangle$ are ensemble averages of the number operators $N_j(t) \equiv \hat{c}_j^\dagger(t)\hat{c}_j(t)$ written in terms of the detected fields: $\hat{c}_j(t) \equiv [\xi_j^- \hat{a}_j(t) + \xi_j^+ \hat{b}_j(t)]$, $[\hat{c}_i(t), \hat{c}_j^\dagger(t)], = \delta_{ij}$, $\xi_j^+ \equiv 2^{-1/2}(\cos\varphi_j + \sin\varphi_j)\exp(i\psi_{j\beta})$, $\xi_j^- \equiv 2^{-1/2}(\cos\varphi_j - \sin\varphi_j)\exp(i\psi_{j\alpha})$, where $\psi_{j\alpha}$, $\psi_{j\beta}$ are phase-shifts induced by the birefringent plate on the fields $\hat{a}_j(t)$, $\hat{b}_j(t)$. $G_j^{(1)}$ shows the expected superposition character of the output field with respect to φ_j and to $\Delta_j^{\pm}\Phi \equiv (\Phi \pm \Psi_j)$: $G_1^{(1)} = \bar{n} + \frac{1}{2}(\bar{n}+1)[1+\cos(2\varphi_1)\cos\Delta_1^-\Phi]$; $G_2^{(1)} = \bar{n} + \frac{1}{2}\bar{n}[1+\cos(2\varphi_2)\cos\Delta_2^+\Phi]$. When the condition of *quantum injection* is not realized the corresponding averages over the input *vacuum state* are: $G_{1,\mathrm{vac}}^{(1)}(\varphi) = G_{2,\mathrm{vac}}^{(1)}(\varphi) = \bar{n}$, for *any* φ or $\Delta\Psi$. By these expressions we obtain the signal-to-noise-ratio related to the S-cat detection: $s/n = 2$, for $\Delta_j^- \Phi = \varphi_j = 0$. The above result immediately suggests a 1st-order $\boldsymbol{\pi}$-interferometric method for S-cat detection on a *single* $\boldsymbol{k}_j$ beam, with *visibility*: $V = (G_{\max}^{(1)} - G_{\min}^{(1)})/(G_{\max}^{(1)} + G_{\min}^{(1)}) = 1$, for $\bar{n} \ll 1$. In Fig. 2 the first realization of the process of *quantum injection* into the *entangled* OPA is reported. In our laboratory experiment, we referred to in Section 2, two equal 1mm thick, BBO crystals are excited by 0.15 picosecond pulses at $\lambda_p = 400$ nm second harmonic generated by a mode-locked Ti : Sa laser at a 76 MHz rep-rate with an average power ≈ 0.3 W. The detection system, consisting of two SPCM silicon avalanche photodetectors connected to an electronic correlator, was equal to the one shown by Fig. 1 inset, but for the absence of the birefringent plate. The phase of the input state was: $\Phi = 0$. The electronic signal, corresponding to the generation by parametric stimulated-emission of $M = 2$ photon couples under injection of the single photon input state (1), was obtained by the experimental determination of $G_2^{(1)}(\varphi)$ and of the related fringe *visibilty*. Consider the two detector device shown in the inset of Fig. 2 and determine the *difference* Δ between the values of the functions $G_2^{(1)}(\varphi)$ and $G_2^{(1)}(\overline{\varphi})$ directly measured by $D_{2\varphi}$ and $D_{2\overline{\varphi}}$. By use of the given expressions for $G_2^{(1)}(\varphi)$ we find: $\Delta = G_2^{(1)}(\varphi) - G_2^{(1)}(\overline{\varphi}) = \bar{n}\cos(2\varphi_2)$ *only if* the space/time superposition within the OPA crystal of the UV laser pulse with the injection pulse is realized, i.e. in condition of *quantum injection*. The pulse superposition is obtained by accurate adjustment of the x-coordinate of the matching 'trombone' shown in Fig. 1. If the quantum injection condition is not realized, the replacement of $G_2^{(1)}(\varphi)$ by $G_{2,\mathrm{vac}}^{(1)}(\varphi) = \bar{n}$ leads to: $\Delta = 0$ for any angle φ. Fig. 2 shows a realization of the quantum injection condition for the π-rotator $R(\varphi)$ set at the angle $\varphi_2 = 0$.

The 2nd-order functions $G_{ij}^{(2)}(x_i, x_j; x_j, x_i) \equiv \langle \Psi_0 | : \hat{N}_i(t)\hat{N}_j(t) : | \Psi_0 \rangle$ are also found: $G_{11}^{(2)} = 2\bar{n}\{\bar{n} + (\bar{n}+1)[1 + \cos(2\varphi_1)\cos\Delta_1^- \Phi]\}$; $G_{22}^{(2)} = 2\bar{n}^2\{1 + [1 + \cos(2\varphi_2)\cos\Delta_2^+ \Phi]\}$; $G_{12}^{(2)} = 2\bar{n}^2 + \bar{n}/2 + \bar{n}[(\bar{n}+1)\cos(2\varphi_1)\cos\Delta_1^- \Phi\ ] + \bar{n}(\bar{n} + 1/2)[1 + \cos(2\varphi_2)\cos\Delta_2^+ \Phi\ ] + \bar{n}(\bar{n} + 1)\{[1 + \cos\Delta\Psi\ ]\cos^2\Delta\varphi^- + [1 - \cos\Delta\Psi\ ]\sin^2\Delta\varphi^+\}$ where: $\Delta\varphi^{\pm} \equiv (\varphi_1 \pm \varphi_2)$; $\Delta\Psi \equiv (\Psi_1 + \Psi_2)$. We may prove, e.g. for all $\Delta_j^{\pm}\Phi = \varphi_j = 0$, that our system realizes the *maximum* quantum mechanical violation of the Cauchy–Schwarz inequality which generally holds in semi-classical field theory: $[g_{12}^{(2)}(0)]^2 \leq g_{11}^{(2)}(0)\, g_{22}^{(2)}(0)$ being: $g_{ij}^{(2)}(0) \equiv G_{ij}^{(2)}(0)[G_i^{(1)}(0)G_j^{(1)}(0)]^{-1}$ [1]. Furthermore, the given expression of $G_{12}^{(2)}$ shows the effects of the *multiparticle* quantum nonseparability and Bell-type nonlocality, contributed by the terms proportional to $\cos(2\varphi_j)$, $\cos\Delta\varphi^{\pm}$. This is a most relevant manifestation of the nonlocality properties of our quantum injected, *entangled* parametric system [14,17].

The absence of *decoherence* within our ideal, non-dissipative multiparticle system is due to its nature of nonlinearly *driven* excitation. As such, it is coupled with a continuously re-phasing environment here provided by the parametric NL polarization. Of course any single photon loss event, mainly contributed in our case by stray reflections, implies an elementary decoherence process. In our laboratory experiment all surfaces were treated by special AR coatings resonant at the working $\lambda = 800$ nm with an overall transmittivity: $T \approx 99.60\%$. This figure implies the loss of a single photon every $\gtrsim 20$ pulses with the generation of $\bar{n} \approx 10$ per pulse. This would make our S-cat experiment quite feasible. A detailed analysis of the decoherence processes shows that the loss of one output photon on either one of the output modes 1, 2 leads to a value of the fringe visibility equal to $V = 0.66$. A comprehensive quantum analysis of the decoherence process due to dissipation shall be reported elsewhere.

In summary, we have given the theory of a novel multiparticle system showing both quantum superposition and quantum entanglement features. This results from a smart interplay of the fundamental paradigms of modern quantum optics, i.e., quadrature-squeezing, multiparticle state-entanglement and quantum-nonseparability in parametric correlations. From a foundational perspective our method allows the first realization of several fundamental nonlocality and noncontestuality tests of quantum mechanics requiring a number of entangled particles larger than two [22].

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