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Excitability of periodic and chaotic attractors in semiconductor lasers with optoelectronic feedback

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Abstract. The dynamics of a semiconductor laser with AC-coupled nonlinear optoelectronic feedback has been experimentally studied. A period doubling sequence of small periodic and chaotic attractors is observed, each of them displaying excitable features. This scenario is found also in a simplified physical model of the system, thus extending the concept of excitability, usually associated to fixed points, also to the case of higher-dimensional attractors.

1 Introduction

Excitability is an important property of a variety of systems studied in many fields, including neural-sciences, chemistry, several branches of physics and engineering, etc. [1–4]. The typical excitable system is characterized by the existence of one stable steady state that can be forced to spike by external stimuli of amplitude above a given, relatively small, threshold. Excitable spikes are associated to a deterministic orbit in the phase space that do not depend on the detailed nature of the stimulus and during which the system is insensitive to any new, above-threshold, perturbation. In two-dimensional (2D) phase-spaces this behaviour can only appear in the vicinity of bifurcations between fixed points and limit cycles, typically Andronov saddle-node collisions and super critical Hopf bifurcations followed by canard explosions. Beyond the bifurcation, the (excitable) steady state is unstable and the system displays self-sustained oscillations. In contrast, higher-dimensional phase-spaces allow more varied and complex dynamics. For instance, the initial Hopf bifurcation can be followed by a period doubling cascade producing a sequence of small periodic and chaotic attractors, that develops before self-oscillations arise. As the mean amplitude of the chaotic attractors grows, a chaotic-spiking regime appears, where large pulses are separated by irregular time intervals in which the system displays small-amplitude chaotic oscillations [5]. This scenario, which is reminiscent of Shil'nikov homoclinic chaos although no homoclinic connection occurs, has been observed in a variety of chemical systems [6–8] and very recently in the dynamics of a semiconductor laser with opto-electronic feedback [9].

Here we show that, in such a system, the chaotic spiking regime can be understood in terms of excitability of a chaotic attractor, where the small chaotic background spontaneously triggers excitable spikes in an erratic but deterministic sequence. The excitable behaviour is observed also on each of the periodic attractors within the period-doubling cascade, thus extending the concept of excitability, usually associated to fixed points, also to the case of higher-dimensional attractors.

2 Experiment

The experimental setup is sketched in Figure 1. We consider a closed-loop optical system, consisting of a single-mode semiconductor laser with AC-coupled nonlinear optoelectronic feedback. The output laser light is sent to a photodetector producing a current proportional to the optical intensity. The corresponding signal is sent to a variable gain amplifier characterized by a nonlinear transfer function of the form $f(w) = Aw/(1 + sw)$, where A is the amplifier gain and s a saturation coefficient, and then fed back to the injection current of the laser. The feedback strength is determined by the amplifier gain, while its high-pass frequency cut-off can be varied (between 1 Hz and 100 KHz) by means of a tunable high-pass filter. The pumping current is typically close to the solitary laser threshold value. External perturbations, in the form of narrow voltage pulses generated by an arbitrary function generator, can be added to the laser pumping current through a mixer.

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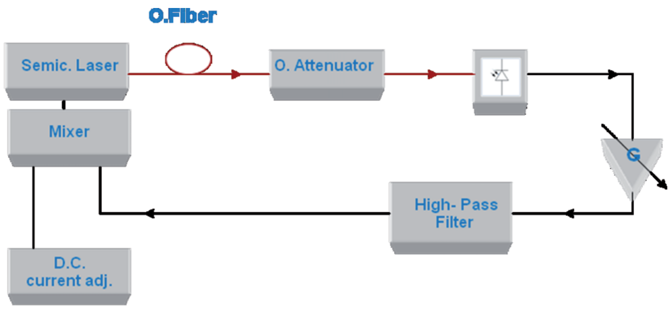

Fig. 1. (Color online) Sketch of the experimental setup.

Fixed the feedback gain and increasing the dc-pumping current a transition from a stable state to a chaotically spiking regime is observed, where large intensity pulses are separated by irregular time intervals in which the system displays small-amplitude chaotic oscillations. Further increase of the current makes the firing rate higher until a periodic regime is eventually reached. The same dynamical sequence can be obtained as the pumping current is kept constant and the amplifier gain is changed. This behaviour has been described in reference [9], where particular attention has been payed to the statistical properties of the irregular inter-spike intervals. Here we mainly focus on the dynamical features of the transitional regime between stable state and chaotic spiking. Fixed the bias current and as the feedback gain is delicately increased we are able to observe part of the bifurcation sequences: the first supercritical Hopf bifurcation of the steady state, the period-2 cycle and the chaotic attractor. The unavoidable intrinsic noise of the system does not allow to further resolve the period-doubling cascade. In each one of this multiplicity of attractors the system is excitable: sufficiently strong stimuli added to the pumping current provoke a long excursion in the phase space, insensitive to the details of the perturbation, before returning to the initial attractor (Fig. 2). During such excursion, the system is refractory to further perturbations as in the fixed point based excitability. On the basis of these results the chaotic spiking regime, observable further increasing the control parameter [9], can be understood in terms of excitability of a chaotic attractor, where the chaotic fluctuations are sufficiently large to eventually trigger an excitable pulse. This results in an erratic, although deterministic, sequence of spikes on top of a chaotic background.

3 Physical model and numerical results

The complete dynamics in our system is ruled by two coupled variables (intensity and population inversion) evolving with two very different characteristic time-scales. The introduction of an AC-feedback optoelectronic loop adds both a third degree of freedom and a third much slower time-scale. The photon density S and carrier density N evolve accordingly to the usual single-mode semiconductor laser rate equations [10] appropriately modified in order

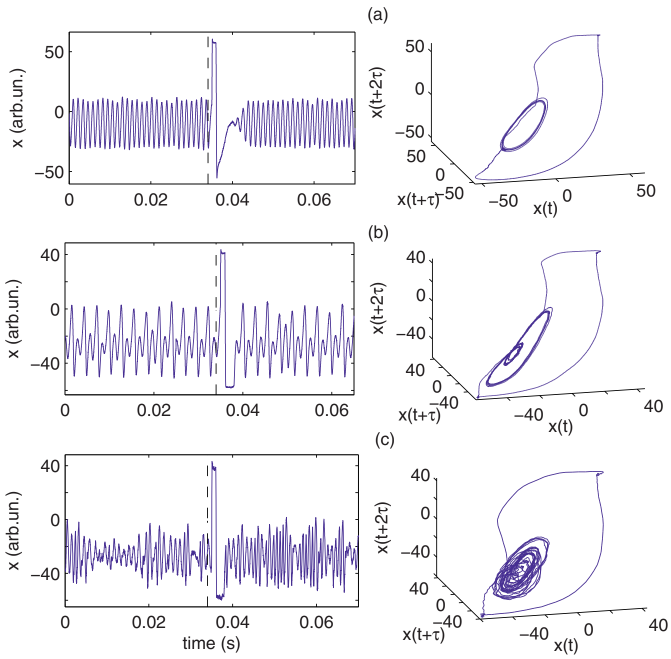

Fig. 2. (Color online) Left panels: Experimental time series for the laser intensity for different parameters. The external perturbation in the form of a sharp pulse (duration 1 ms) has been added to the pumping current at time marked by the vertical dashed line. Right panels: Reconstructed phase space for the laser intensity by embedding method.

to include the AC-coupled feedback loop

$$\dot{S} = [g(N - N_t) - \gamma_0] S$$
$$\dot{N} = \frac{I_0 + f_F(I)}{eV} - \gamma_c N - g(N - N_t) S$$
$$\dot{I} = -\gamma_f I + k\dot{S} \qquad (1)$$

where I is the high-pass filtered feedback current (before the nonlinear amplifier), $f_F(I) \equiv AI/(1+s'I)$ is the feedback amplifier function, I_0 is the bias current, e the electron charge, V is the active layer volume, g is the differential gain, N_t is the carrier density at transparency, γ_0 and γ_c are the photon damping and population relaxation rate, respectively, γ_f is the cutoff frequency of the high-pass filter and k is a coefficient proportional to the photodetector responsivity. By introducing the new variables $x = \frac{g}{\gamma_c}S$, $y = \frac{g}{\gamma_0}(N - N_t)$, $w = \frac{g}{k\gamma_c}I - x$ and the time scale $t' = \gamma_0 t$. The rate equations then become

$$\dot{x} = x(y - 1) \qquad (2)$$
$$\dot{y} = \gamma(\delta_0 - y + f(w + x) - xy) \qquad (3)$$
$$\dot{w} = -\epsilon(w + x) \qquad (4)$$

where $f(w + x) \equiv \alpha\frac{w+x}{1+s(w+x)}$, $\delta_0 = (I_0 - I_t)/(I_{th} - I_t)$ ($I_{th} = eV\gamma_c(\frac{\gamma_0}{g} + N_t)$ is the solitary laser threshold current), $\gamma = \gamma_c/\gamma_0$, $\varepsilon = \omega_0/\gamma_0$, $\alpha = Ak/(eV\gamma_0)$ and $s = \gamma_c s' k/g$. The blow-up of large phase-space orbits

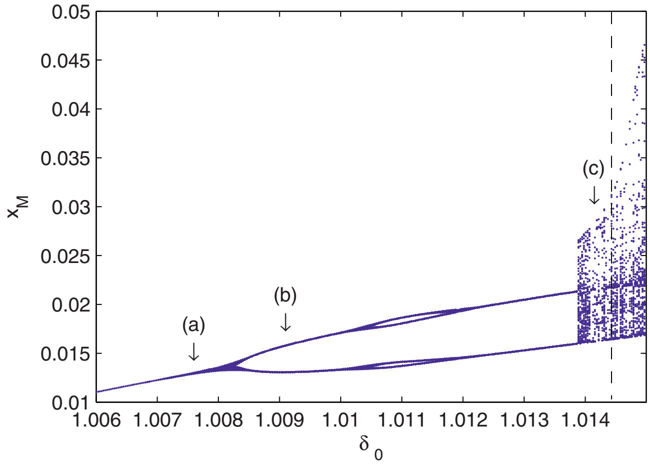

Fig. 3. (Color online) Bifurcation diagram for a model equations. The parameter δ_0 used in Figures 4a–4c is marked with an arrow. The horizontal dashed line separates the regime without (to the left) and with (to the right) large spikes. The system parameters are: $s = 11$, $\alpha = 1$, $\gamma = 0.001$ and $\epsilon = 2 \times 10^{-5}$.

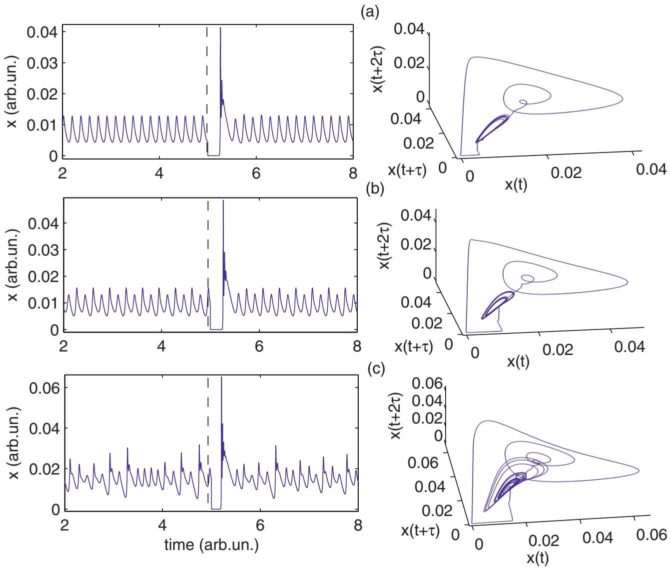

Fig. 4. Numerical simulations of the model equations. Left panels: time series for the laser intensity. The excitation pulse is added to the parameter δ_0 at the time marked by the vertical dashed line. The duration of the external pulse is smaller than the refractory period of the excitable pulse. Right panels: reconstructed phase space. The control parameter has a value: (a) $\delta_0 = 1.0075$, (b) $\delta_0 = 1.009$ and (c) $\delta_0 = 1.01405$. The other system parameters are: $s = 11$, $\alpha = 1$, $\gamma = 0.001$ and $\epsilon = 2 \times 10^{-5}$.

and the occurrence of a chaotic spiking regime can be understood in terms of the geometric theory of singular perturbation (see Ref. [9]). Here we will just discuss the numerical results in comparison to the experimental observations. Numerical simulations have been done with the use of fourth-order Runge-Kutta integration scheme, with time-step $dt = 0.01$. The total simulation time chosen depends strongly on the magnitude of the temporal scales defined by the parameters γ and ϵ. Between the steady state and the chaotic spiking the system passes through a cascade of period doubled and chaotic attractors of small amplitude. This is illustrated by Figure 3 where a bifurcation diagram is computed from our system varying δ_0 over a small interval contiguous to the initial Hopf bifurcation. As observed in the numerical simulations, on each of these attractors the system is excitable (see Fig. 4). Sufficiently strong perturbations added to the parameter δ_0 force the system to respond with a pulse that do not depend on the detailed nature of the stimulus and during which the system is insensitive to any new perturbation.

4 Conclusions

In this work, we have studied experimentally and theoretically the dynamics of a semiconductor laser with AC-coupled nonlinear optoelectronic feedback. A period doubling sequence of small periodic and chaotic attractors is observed, each of them displaying excitable features. On one side, these results extend the fixed point based excitability concept also to the case of higher-dimensional attractors. On the other, they allow the interpretation of the chaotically spiking regime in terms of excitability of a chaotic attractor, where the small chaotic background spontaneously triggers excitable spikes in an erratic but deterministic sequence. The transitions between chaotic and periodic mixed mode oscillation, experimentally observed in our system and currently under investigation, could be associated with the transition from chaotic to periodic windows in the underlying Feigenbaum cascade of the small amplitude background attractor.

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