

## Quantum Decoherence Reduction by Increasing the Thermal Bath Temperature

A. Montina and F. T. Arecchi

*Dipartimento di Fisica, Università di Firenze, Via Sansone 1, 50019 Sesto Fiorentino (FI), Italy*  
(Received 17 September 2007; published 25 March 2008)

The well-known increase of the decoherence rate with the temperature, for a quantum system coupled to a linear thermal bath, no longer holds for a different bath dynamics. This is shown by means of a simple classical nonlinear bath, as well as a quantum spin-boson model. The anomalous effect is due to the temperature dependence of the bath spectral profile. In the case of the second model, a link with the quantum Zeno effect is provided. The decoherence reduction via the temperature increase can be relevant for the design of quantum computers.

DOI: [10.1103/PhysRevLett.100.120401](https://doi.org/10.1103/PhysRevLett.100.120401)

PACS numbers: 03.67.Pp, 03.65.Yz

The rapid development of nanotechnology has opened the possibility of assembling materials and devices at the atomic scale, such as nanomaterials and nanoelectromechanical systems [1]. Because of quantum mechanical and statistical effects, the properties of these systems can be considerably different from their macroscopic counterpart. Electrons in nanofabricated quantum dots as well as nanomechanical devices are possible candidates for quantum bits at nanometer scale, as suggested by recent experiments [2]. Quantum computers could take advantage of quantum superpositions in order to perform highly efficient parallel calculations. The main difficulty in the implementation is represented by decoherence, due to the coupling with the environment [3]. This effect increases with the number of quantum bits and is devastating even with a small thermal bath coupling. Thus, in order to reduce the decoherence of quantum superpositions, a sufficiently small thermal disturbance and environment coupling are required. The thermal disturbance is expected to be reduced by decreasing the bath temperature. This is true when the quantum system is coupled with a high number of degrees of freedom and the bath can be described as a linear noise source. On the contrary, at the nanometer scale, the quantum system can be directly coupled to few modes, whose power spectrum does not necessarily grow with temperature. Thus, it is possible to have a reversed effect, i.e., the reduction of decoherence by increasing the thermal bath temperature.

A similar counterintuitive behavior occurs, for example, in thermal rectifiers [4], where the heat flux can be suppressed by increasing the temperature difference of the junctions. The rectifier consists of two or three mutually coupled layers, whose phononic bands depend on the temperature via nonlinear effects. As the ends of the chain are coupled with two heat baths at different temperatures  $T_1$  and  $T_2$ , the overlaps among the phononic bands in each layer become functions of  $T_1$  and  $T_2$  and are not identical when the thermal baths are exchanged. As a consequence, the thermal flux can change when the rectifier is reversed.

In this Letter, we show that the decoherence rate of a quantum system coupled to a nonlinear thermal bath can decrease with the temperature. As in thermal rectifiers, this

effect is due to the nonlinearity, which makes the shape of the bath spectrum dependent on the temperature. For our purpose, we consider a classical nonlinear bath model, consisting of a one-dimensional particle in a double-well potential, strongly coupled to a linear bath. Furthermore, we also consider a full quantum mechanical bath model, consisting of a boson bath coupled to a two-state system, which is the low temperature limit of a double-well system under suitable conditions [5]. A probe quantum system is coupled to the nonlinear bath and its decoherence rate is evaluated. For this last model, we will show that the decoherence suppression is related to the quantum Zeno effect (QZE) [6].

The equation of motion of a one-dimensional particle coupled to a white noise thermal bath is  $m\ddot{x} = -V'(x) - m\bar{\gamma}\dot{x} + \sqrt{2D}\eta(t)$ , where  $m$ ,  $V(x)$ ,  $\bar{\gamma}$ , and  $D$  are the particle mass, the external potential, the dissipation, and diffusion coefficients, respectively. The function  $\eta(t)$  is a Gaussian noise with  $\langle \eta(t)\eta(t') \rangle = \delta(t-t')$ . The fluctuation-dissipation theorem establishes that  $D = \bar{\gamma}mk_bT$ ,  $k_b$  and  $T$  being the Boltzmann constant and the thermal bath temperature, respectively. We consider the double-well potential  $V(x) = -ax^2/2 + bx^4/4$ . By means of the rescaling  $x \rightarrow (a/b)^{1/2}x$ ,  $t \rightarrow (m/a)^{1/2}t$ , and  $T \rightarrow \frac{a^2}{bk_b}T$ , the equation of motion becomes

$$\ddot{x} = x - x^3 - \gamma_1\dot{x} + \sqrt{2\gamma_1 T}\eta(t), \quad (1)$$

where  $\gamma_1 \equiv \bar{\gamma}(m/a)^{1/2}$ . The independent parameters are the rescaled dissipation coefficient and temperature.

At finite temperature, the particle has a finite probability to jump from a well to the other one. The hopping rate is [7]  $R \equiv (\sqrt{2\pi\gamma_1})^{-1} \exp[-(4\gamma_1 T)^{-1}]$  (Kramers rate). In the limit of very low temperature, the hopping rate becomes very small and the particle remains in one well for a large time. As a consequence, the spectrum of  $x(t)$  has a high zero-frequency peak. As the temperature is increased, the hopping rate between the two wells grows, the central peak of the spectrum reduces, and two smooth peaks around the frequencies  $\pm \pi R$  are generated. This behavior is shown in Fig. 1, where we report the spectrum  $I(\omega, T) = 2\text{Re} \int_0^\infty \langle x(0)x(t) \rangle_T e^{i\omega t} dt$ , for  $\gamma_1 = 0.4$  and some values of

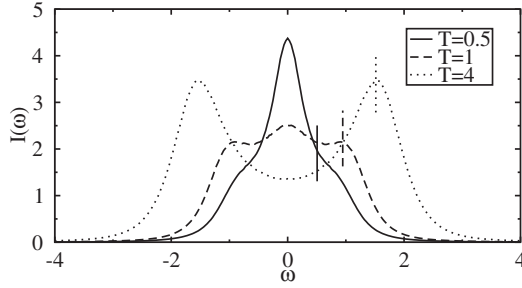


FIG. 1. Power spectrum of  $x(t)$  for some values of  $T$ . The vertical lines are placed at  $\pi R$  for each spectrum.

the temperature.  $I(\omega, T)$  has been evaluated numerically over a time interval of 200, in the nondimensional units, and 50 000 ensemble realizations. We have verified that the time interval is sufficiently large to ensure a negligible time cutoff of the integral. In Fig. 2, we plot  $I(0, T)$  (solid line) and  $I(0, T)/T \equiv K(0, T)$  (dashed line) as functions of  $T$ . The decreasing behavior of  $K(0, T)$  suggests a simple model of thermal rectifier, as will be discussed in an extended paper. Here we will use the decreasing behavior of  $I(0, T)$  to show the anomalous behavior of decoherence induced by the nonlinearity. We consider a two-state quantum system ( $A$ ) coupled to the double-well oscillator ( $B$ ), as sketched in Fig. 3. The overall system  $B$ - (linear bath) is the nonlinear bath seen by  $A$ . The linear bath and the anharmonicity of  $B$  guarantee that the overall system is a reservoir and a nonlinear system, respectively. We assume that the system  $A$  is not directly coupled to the linear bath. Such a coupling would be relevant in the high-temperature regime.

This classical bath model is suitable for a sufficiently high thermal bath temperature  $T$ . A full quantum approach will be introduced later on. We consider an interaction Hamiltonian linear in  $x$  and assume that the quantum state  $|\psi\rangle \equiv \psi_{-1}|-1\rangle + \psi_1|1\rangle$  satisfies the Schrödinger equation  $i\frac{\partial|\psi\rangle}{\partial t} = [(\Omega/2)\hat{\sigma}_A^1 + \epsilon\hat{\sigma}_A^3 x(t)]|\psi\rangle$ , where  $\hat{\sigma}_A^i$  are the Pauli matrices in the orthonormal basis  $\{|-1\rangle, |1\rangle\}$ .  $\Omega$  is the energy difference between the bare energy levels  $[|1\rangle \pm |-1\rangle]/\sqrt{2}$  and  $\epsilon$  is the coupling coefficient between the quantum and classical systems. The variable  $x$  satisfies the classical Eq. (1). We have seen that the spectrum  $I(\omega, T)$  of

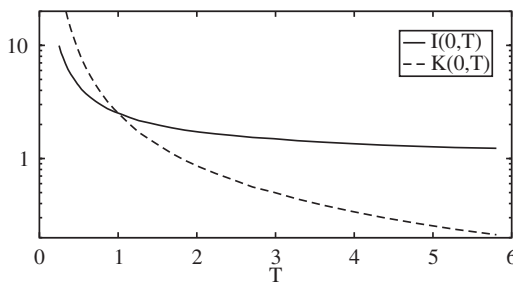


FIG. 2.  $I(0, T)$  and  $K(0, T)$  as functions of the temperature, in logarithmic scale.

the anharmonic oscillator is a decreasing function of temperature only for sufficiently small values of  $|\omega|$ . If the quantum system has a dynamics sufficiently slower than the classical hopping rate ( $\Omega \ll R$ ), then it is sensitive only to the low frequency components of the bath spectrum. We satisfy the condition  $\Omega \ll R$  by assuming that the two levels are nearly degenerate. For the sake of simplicity we set  $\Omega = 0$ . If  $|\psi(0)\rangle = (|-1\rangle + |1\rangle)/\sqrt{2}$  is the quantum state at time  $t = 0$ , then at a subsequent time  $t$  the density matrix associated with a single realization of  $x(t)$  is

$$\hat{\rho}[x](t) = \frac{1}{2} \begin{pmatrix} 1 & e^{-2i\epsilon \int_0^t x(t)dt} \\ e^{2i\epsilon \int_0^t x(t)dt} & 1 \end{pmatrix},$$

and it is a functional of the thermal bath variable. This dependence is removed by averaging over every realization of  $x(t)$ . The reduced density matrix is  $\langle \hat{\rho}[x] \rangle_x \equiv \hat{\rho}_A$ , and the modulus of its off-diagonal elements

$$\left| \left\langle e^{2i\epsilon \int_0^t x(t)dt} \right\rangle \right| \equiv C(t) \quad (2)$$

is a measure of the coherence between the superposed states  $|-1\rangle$  and  $|1\rangle$ .  $C$  is a good indicator since  $|-1\rangle$  and  $|1\rangle$  are two perfect pointer states of a pure dephasing process. It ranges between the values 0 (no coherence) and 1 (complete coherence).

We will show that the decoherence rate of  $A$  is proportional to  $I(0, T)$ , provided that  $\epsilon$  is sufficiently small. Let  $t_c$  be the correlation time of  $x(t)$  and  $\delta t \gg t_c$  be a finite time interval. Let us consider the finite difference  $C(t + \delta t) - C(t) = C(t)[\langle e^{2i\epsilon \int_t^{t+\delta t} x(t)dt} \rangle - 1]$ . For a large temperature range, we can assume  $\langle x^2 \rangle \approx 1$ ; thus, if  $2\epsilon\delta t \ll 1$ , then we can approximate the previous equation as  $C(t + \delta t) - C(t) \approx -2\epsilon^2 C(t) \int_t^{t+\delta t} dt' \int_t^{t+\delta t} d\bar{t} \langle x(t')x(\bar{t}) \rangle$ . Since  $\delta t \gg t_c$ , we can replace the second integral with  $I(0, T)$  and have  $\dot{C} = -2\epsilon^2 I(0, T)C \equiv -D(T)C$ . The decoherence rate  $D(T)$ , being proportional to  $I(0, T)$ , is a decreasing function of  $T$  (see Fig. 2). This equation is correct when the integration step  $\delta t$  is much smaller than  $1/(2\epsilon)$  and much larger than  $t_c$ ; thus, the condition  $1/t_c \gg 2\epsilon$  has to be satisfied. For  $\gamma_1 = 0.4$ , the correlation time is about 10 at  $T \sim 0.25$  and decreases for higher temperatures. At  $T \sim 1-2$ ,  $t_c$  is, for example, about 5. Thus,  $\epsilon$  has to be smaller than about 0.1–0.05. This classical model demonstrates that the quantum decoherence rate is a decreasing function of the temperature when the quantum system is resonant with bath spectral components depleted by the thermal

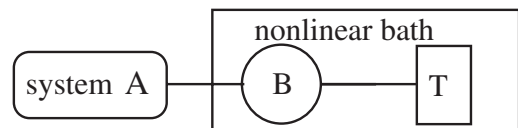


FIG. 3. Schematic representation of the nonlinear thermal bath, coupled to the quantum system  $A$ .  $B$  is the anharmonic oscillator.

energy supplying. The nonlinearity is a fundamental ingredient, since for linear baths the spectrum grows linearly with temperature at any frequency.

Our classical model works when high order energy levels of the double-well system are activated, i.e., for a sufficiently high temperature  $T$ . Now, we consider the opposite limit and assume that only the two lowest energy states are populated. This is possible if the energy difference  $\Delta$  between the ground and first excited states is much smaller than the higher transition energies.  $\Delta$  is the quantum tunneling frequency between the two wells. Let the Pauli matrix  $\hat{\sigma}_B^3$  be the position operator, whose eigenstates  $|\pm 1\rangle$  correspond to the particle in the left or right well. This quantum system ( $B$ ) is coupled to a boson bath (spin-boson model) and to the two-state probe system ( $A$ ) described by the Pauli matrices  $\hat{\sigma}_A^i$ . The overall system  $B$ - (boson bath) is the nonlinear thermal bath of  $A$  (Fig. 3).

More precisely, we study a model with the Hamiltonian  $\hat{H} = \frac{\Omega}{2}\hat{\sigma}_A^1 + \frac{\Delta}{2}\hat{\sigma}_B^1 + \epsilon\hat{\sigma}_A^3\hat{\sigma}_B^3 + \sum_k[\eta\hat{\sigma}_B^3(\hat{a}_k + \hat{a}_k^\dagger) + \omega_k\hat{a}_k^\dagger\hat{a}_k]$ , where  $\hat{a}_k$  and  $\hat{a}_k^\dagger$  are the annihilation and creation operators of the bath oscillators. In the present model  $x$  is replaced by the Pauli matrix  $\hat{\sigma}_B^3$ . In order to understand how the decoherence of  $A$  depends on its bare frequency  $\Omega$ , we have to evaluate the two-time correlation function  $\langle\hat{\sigma}_B^3(0)\hat{\sigma}_B^3(t)\rangle$  in the Heisenberg representation for  $\epsilon = 0$ , as done with the first model. For  $\eta$  sufficiently small (see below for a more precise condition), we can use the rotating-wave approximation and find the master equation for  $B$  (section 10.5 of Ref. [8])

$$\frac{\partial\hat{\rho}_B}{\partial t} = -i\frac{\Delta}{2}[\sigma_B^1, \hat{\rho}_B] + \mathcal{L}\hat{\rho}_B, \quad (3)$$

where

$$\begin{aligned} \mathcal{L}\rho_B \equiv & -\gamma_b\bar{n}[\hat{\sigma}_B^-\hat{\sigma}_B^+\hat{\rho}_B + \hat{\rho}_B\hat{\sigma}_B^-\hat{\sigma}_B^+ - 2\hat{\sigma}_B^+\hat{\rho}_B\hat{\sigma}_B^-] \\ & -\gamma_b(\bar{n}+1)[\hat{\sigma}_B^+\hat{\sigma}_B^-\hat{\rho}_B + \hat{\rho}_B\hat{\sigma}_B^+\hat{\sigma}_B^- - 2\hat{\sigma}_B^-\hat{\rho}_B\hat{\sigma}_B^+] \end{aligned} \quad (4)$$

and  $\hat{\sigma}_B^\pm \equiv (\hat{\sigma}_B^2 \pm i\hat{\sigma}_B^3)/2$ .  $\gamma_b$  is a constant coefficient that depends on  $\eta$  and the density of the boson modes at the frequency  $\omega_k = \Delta$  and  $\bar{n}(T) \equiv [\exp(\Delta/T) - 1]^{-1}$  is their expectation value  $\langle\hat{a}_k^\dagger\hat{a}_k\rangle$ . This equation is well known since it describes the decay of a two-level atom coupled to optical modes.

From the master Eq. (3) and using the quantum regression theorem, we have that the correlation function at equilibrium is [8]

$$\langle\hat{\sigma}_B^3(0)\hat{\sigma}_B^3(t)\rangle = e^{-|t|/\tau_c(T)}[e^{-i\Delta t}n_\uparrow + e^{i\Delta t}n_\downarrow], \quad (5)$$

where  $n_\uparrow \equiv \langle\hat{\sigma}_B^+\hat{\sigma}_B^-\rangle = \frac{e^{-\Delta/2T}}{2\cosh(\Delta/2T)}$  and  $n_\downarrow \equiv \langle\hat{\sigma}_B^-\hat{\sigma}_B^+\rangle = 1 - n_\uparrow$  are the probability weights for the energy levels of  $B$  and  $\tau_c(T) \equiv [(2\bar{n}+1)\gamma_b]^{-1}$  is the temperature-dependent correlation time. The corresponding spectral function is  $I(\omega, T) = \frac{2\tau_c n_\uparrow}{1+\tau_c^2(\omega-\Delta)^2} + \frac{2\tau_c n_\downarrow}{1+\tau_c^2(\omega+\Delta)^2}$ , which has two peaks centered in  $\pm\Delta$ , whose areas and widths are

constant and linear functions of temperature, respectively. The rotating-wave approximation is suitable when the two peaks have a negligible overlap, i.e., when  $\Delta \gg 1/\tau_c$ . It is clear that for  $\omega \neq \Delta$ , the spectrum initially increases with temperature, reaches a maximum, then decreases and tends to zero for  $T \rightarrow \infty$ . In the particular case  $\omega = \Delta$ ,  $I(\omega, T)$  is a monotonic decreasing function for any value of temperature. With this in mind, we choose the frequency  $\Omega$  of system  $A$  in order to maximize the effect of temperature on the decoherence reduction. The system  $A$  has to be resonant with the part of the spectrum  $I(\omega, T)$ , which is strongly depleted by the temperature increase; thus, we assume  $\Omega = \Delta$ . This case simplifies the analysis and is sufficient for our purposes.

At this point, we take  $\epsilon \neq 0$  and write out the master equation for  $A$  and  $B$ . If  $\epsilon \ll 1/\tau_c$ , then it is obtained from Eq. (3) by adding the bare Hamiltonian of  $A$  and the interaction Hamiltonian  $\epsilon\hat{\sigma}_A^3\hat{\sigma}_B^3$  to the bare Hamiltonian of  $B$ . In the rotating-wave representation (interaction picture), the overall bare Hamiltonian  $(\Delta\hat{\sigma}_A^1 + \Delta\hat{\sigma}_B^1)/2$  is removed and the following replacements are performed:  $\hat{\sigma}_{A,B}^3 \rightarrow \hat{\sigma}_{A,B}^-e^{-i\Delta t} + \hat{\sigma}_{A,B}^+e^{i\Delta t}$ . The condition  $\epsilon \ll 1/\tau_c$  justifies a rotating-wave approximation also for the interaction between  $A$  and  $B$ . Thus, we obtain the master equation

$$\frac{\partial\hat{\rho}}{\partial t} = -i\epsilon[\hat{\sigma}_A^+\hat{\sigma}_B^- + \hat{\sigma}_A^-\hat{\sigma}_B^+, \hat{\rho}] + \mathcal{L}\hat{\rho}. \quad (6)$$

In order to eliminate the parameter  $\Delta$  in  $\bar{n}$ , we define the rescaled temperature  $\tilde{T} \equiv T/\Delta$ .

In this model, the vectors  $|1\rangle$  and  $|-1\rangle$  are not perfect pointer states because of the presence of the bare Hamiltonian of the system  $A$ ; thus, we cannot use the previous definition of coherence. We need a generalized measure of the deviation from the unitary evolution. A good indicator is provided by the quantity  $\mathcal{C} = \sqrt{2\text{Tr}[\hat{\rho}_A^2] - 1}$ ,  $\hat{\rho}_A$  being the reduced density operator of the system  $A$ . For pure states it is equal to 1 and tends to zero for completely incoherent high-temperature thermal states. It is easy to prove that this definition is equivalent to Eq. (2) when  $|1\rangle$  and  $|-1\rangle$  are equally populated pointer states. At thermal equilibrium, we have  $\mathcal{C} = \tanh[1/(2\tilde{T})]$ . Obviously, it is a decreasing function of temperature and for  $\tilde{T} \rightarrow 0, \infty$  takes the maximal and minimal values 1 and 0, respectively. The transient behavior of  $\mathcal{C}$  is numerically evaluated by setting the probe system  $A$  in the pure state  $[|-1\rangle + |1\rangle]/\sqrt{2}$  at the initial time  $t = 0$ , with the system  $B$  thermalized at the temperature  $\tilde{T}$ . In Fig. 4 we report  $\mathcal{C}(t)$  as a function of time for  $\gamma_b = 1$ ,  $\epsilon = 5 \times 10^{-2}$ , and some values of  $\tilde{T}$ . Note that, for  $\tilde{T} = 80$ , the decoherence rate becomes very small with respect to the value at low temperature.

The master Eq. (6) has been obtained by means of the conditions  $\epsilon \ll 1/\tau_c \ll \Delta$ . Under the constraint  $\epsilon \ll 1/\tau_c$ , it is possible to find an analytical relation, since some components of  $\hat{\rho}(t)$  can be adiabatically eliminated.

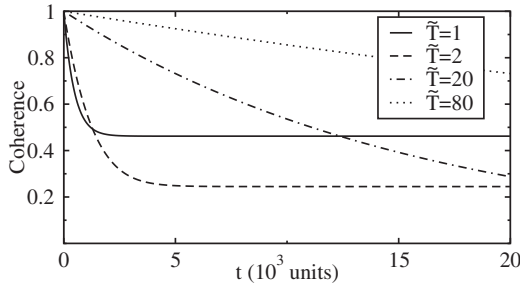


FIG. 4. Numerically evaluated coherence  $\mathcal{C}$  as a function of time for some values of temperature  $\tilde{T}$ .

The following approximate solution for the reduced density operator of the probe system holds:

$$\hat{\rho}_A = \hat{\rho}_{th} + e^{-\epsilon^2 \tau_c t} (c_1 \hat{\sigma}_A^2 + c_3 \hat{\sigma}_A^3) + e^{-2\epsilon^2 \tau_c t} c_2 \hat{\sigma}_A^1, \quad (7)$$

where  $\hat{\rho}_{th}$  is the thermalized density operator,  $c_k$  are constant parameters that depend on the initial condition, and  $\epsilon^2 \tau_c = \epsilon^2 \gamma_c^{-1} \tanh(1/2\tilde{T}) \equiv \Gamma_d$  is a decreasing function of temperature. Notice that this solution could be obtained directly by the correlation function (5). By means of the rotating-wave approximation, the standard hypotheses used to derive Eq. (3), and the result of the previous model (second order expansion in the coupling strength and Markov approximation), one finds that  $\hat{\rho}_A$  is the solution of a master equation identical to Eq. (3), but with  $\gamma_b$  replaced by  $\epsilon^2 \tau_c \tanh(1/2\tilde{T}) = \epsilon^2 \gamma_c^{-1} \tanh^2(1/2\tilde{T})$ . Equation (7) is its exact solution. This derivation will be discussed in an expanded paper.

As in the classical model, we find that the decoherence and relaxation rates are decreasing functions of temperature. In this case the effect is much stronger. For  $\tilde{T} \gg 1$ , the decoherence rate is inversely proportional to the temperature, and for a sufficiently high thermal noise it can be practically reduced to zero. The other interesting feature is the reduction of  $\Gamma_d$  by increasing the coupling parameter  $\gamma_b$ . These effects are due to the depletion of bath spectrum  $I(\omega, T)$  at the frequencies resonant with the quantum system.

There is an interesting link between the anomalous behaviors in the last model and the QZE [6]. Assume that the two systems  $A$  and  $B$  are in the state  $|{-1}\rangle_A |1\rangle_B$  at time  $t$ . Because of their mutual coupling, at a subsequent time  $t + \Delta t$ , they evolve to the state  $|{-1}\rangle_A |1\rangle_B + \epsilon \Delta t |1\rangle_A |{-1}\rangle_B$  and the probability of the transition  $|{-1}\rangle_A |1\rangle_B \rightarrow |1\rangle_A |{-1}\rangle_B$  is  $(\epsilon \Delta t)^2$ . In the meantime, the coherence of this superposition is destroyed by the thermal bath, which acts as an observer of the pointer states  $|{-1}\rangle_B$  and  $|1\rangle_B$ . Thus, if  $\tau_c$  is the correlation time of  $B$ , the probability of the transition after a time  $\Delta t_0 \gg \tau_c$  is  $P_{(-1,1) \rightarrow (1,-1)} \simeq [\epsilon \tau_c]^2 \frac{\Delta t_0}{\tau_c} = \epsilon^2 \tau_c \Delta t_0$  and we obtain heuristically the relaxation rate in Eq. (7). Alternatively, the anomalous effect can be explained as due to a rapid change of the phases of  $|{-1}\rangle_A |1\rangle_B$  and  $|1\rangle_A |{-1}\rangle_B$ , which reduces

the coherence of the two states and consequently their transition probability. It is interesting to note that the decoherence rate suppression for  $T \rightarrow \infty$  in the last model can be obtained by a condition established by Pascazio [9] [equation (25) in that paper], although in our model the overall system  $A - B$  has not perfect pointer subspaces, as in the case of Ref. [9]. By means of this condition one evaluates the Zeno subspaces [10] and find that the Hilbert space of  $A$  is decoherence-free, provided that  $\bar{n} \gg 1$ .

In conclusion, we have considered two examples of nonlinear baths weakly coupled to a quantum system  $A$  and shown that the decoherence rate of  $A$  is a monotonic decreasing function of temperature. This anomalous behavior is due to the temperature dependence of the spectral profile in nonlinear baths. It is important to realize that at a microscopic scale nonlinear effects can be relevant, as it occurs with thermal rectifiers. For more realistic models, the decoherence rate could not be a monotonic decreasing function of temperature. For example, if the system  $A$  is directly coupled also to the linear bath, then its decoherence rate grows in the high-temperature regime. However our analysis indicates that the decoherence rate is not necessarily minimal at  $T = 0$ , as for linear baths. Decoherence reduction by the thermal energy increase can have relevant consequences for the design of quantum computers at nanometer scale. An experimental test of this effect is feasible with the present technology and can be observed in nanometer devices [2,11]. For example,  $A$  and  $B$  could be a superconducting qubit and a microwave resonator [11].

This work was supported by Ente Cassa di Risparmio di Firenze under the project ‘‘Dinamiche cerebrali caotiche.’’

- 
- [1] M. Blencowe, Phys. Rep. **395**, 159 (2004).
  - [2] C.P. Garcıa *et al.*, Phys. Rev. Lett. **95**, 266806 (2005); M.D. LaHaye, O. Buu, B. Camarota, and K.C. Schwab, Science **304**, 74 (2004); A. Haik *et al.*, Nature (London) **443**, 193 (2006).
  - [3] W.H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
  - [4] M. Terraneo, M. Peyrard, and G. Casati, Phys. Rev. Lett. **88**, 094302 (2002).
  - [5] A.J. Leggett, S. Chakravarty, A.T. Dorsey, Matthew P. Fisher, Anupam Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
  - [6] B. Misra and E.C.G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
  - [7] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
  - [8] D.F. Walls and G.J. Milburn, *Quantum Optics* (Spindler, New York, 1995).
  - [9] S. Pascazio, J. Mod. Opt. **51**, 925 (2004).
  - [10] P. Facchi and S. Pascazio, Phys. Rev. Lett. **89**, 080401 (2002).
  - [11] A. Wallraff *et al.*, Nature (London) **431**, 162 (2004).