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Upper limits of signal to noise ratio has been theoretically calculated for charge measuring amplifiers as used in connection with radiation detectors taking into account the additional constraint of a given resolving time.

Signal to noise ratio and resolving time in pulse amplifiers for nuclear detectors

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INTRODUCTION

The problem of measuring the charge Q corresponding to a current pulse of known shape applied to the input capacity C of an amplifier has been largely dealt with and the results are critically examined and exposed in a paper of E. Baldinger and W. Franzen [1].

Let $y = A_0 \cdot s_0(t)$ be the voltage signal produced at the input of the amplifier by the current pulse; $s_0(t)$ is a known time function and $A_0 = Q/C$ is the quantity to be measured in presence of a (reduced to the input) voltage noise having power spectrum $N_0(\omega)$. With an « optimum » linear network synthesized from the knowledge of $s_0(t)$ and $N_0(\omega)$, the best signal to noise ratio (ratio of the peak amplitude of the output pulse to the r.m.s. noise voltage) becomes:

$$\eta_\infty = 2 A_0 \left| \int_0^\infty \frac{|S_0(\omega)|^2}{N_0(\omega)} \, \mathrm{d}f \right|^{1/2} \qquad (1)$$

where $S_0(\omega)$ is the Fourier spectrum of $s_0(t)$.

The optimum network is composed by two cascaded networks 1 and 2.

Network 1 transforms the noise spectrum $N_0(\omega)$ into a white one N_1; at the same time network 1 transforms the signal $A_0 s_0(t)$ into $A_1 s_1(t)$.

Network 2 has a delta pulse response which is the mirror image of $s_1(t)$ with respect to a time T_m which defines the used part of $s_1(t)$. η given by (1) corresponds to $T_m \to \infty$, that is to the use of the whole $s_1(t)$.

If a part of $s_1(t)$ is used, the signal to noise ratio is reduced to:

$$\eta = K\, \eta_\infty$$

where

$$K = \frac{\int_0^{T_m} s_2^1(t)\, dt}{\int_0^{\infty} s_2^1(t)\, dt} \qquad (2)$$

By this theory, even for a finite T_m, no limit is put to the width of the output pulse $s_2(t)$.

In this work we consider the same problem with the constraint of a given resolving time T_2, so that the output pulse $s_2(t)$ should be identically zero outside a time interval T_2.

OPTIMUM NETWORK SYNTHESIS

Also in our case the network is synthesized in two steps. A network 1 identical to that mentioned in the introduction reduces the problem to the consideration of a white noise spectrum N_1 and a signal $A_1 s_1(t)$.

Let us assume $s_1(t)$ to get an exponential $e^{-t/\sigma}$ for $t > t_1$, within a given accuracy (dominant real pole) (fig. 1).

We require that the output of network 2 fed by the signal $s_1(t)$ should be identically zero after a time T_2, where $T_2 > t_1$.

This requirement is equivalent to the following two conditions:

1. The delta pulse response $f(t)$ of network 2 must be identically zero for $t > T_2 - t_1 = \lambda$ in order that no contribution is given to $s_2(t)$ by the $(0, t_1)$ part of $s_1(t)$, after T_2.

$$f(t) \equiv 0 \qquad t > \lambda. \qquad (3)$$

2. The exponential tail $e^{-t/\sigma}$ must give no contribution to $s_2(t)$ after T_2, that is:

$$\int_0^{\lambda} f(\tau)\, e^{-(t-\tau)/\sigma}\, d\tau \equiv 0 \text{ or } \int_0^{\lambda} e^{\tau/\sigma} f(\tau)\, d\tau = 0 \qquad (4)$$

Condition (4) is equivalent to state that the transfer function of network 2 has a zero coincident with the dominant pole of the $s_1(t)$ Laplace transform.

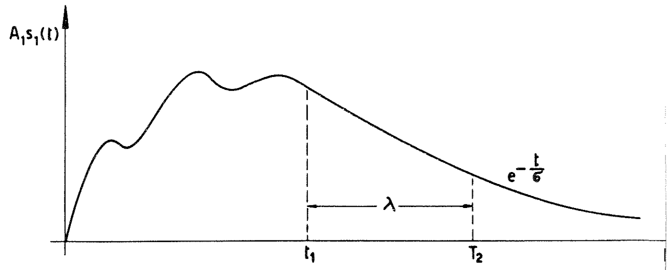

Fig. 1 - Input voltage waveform of network 2: tail is assumed exponential.

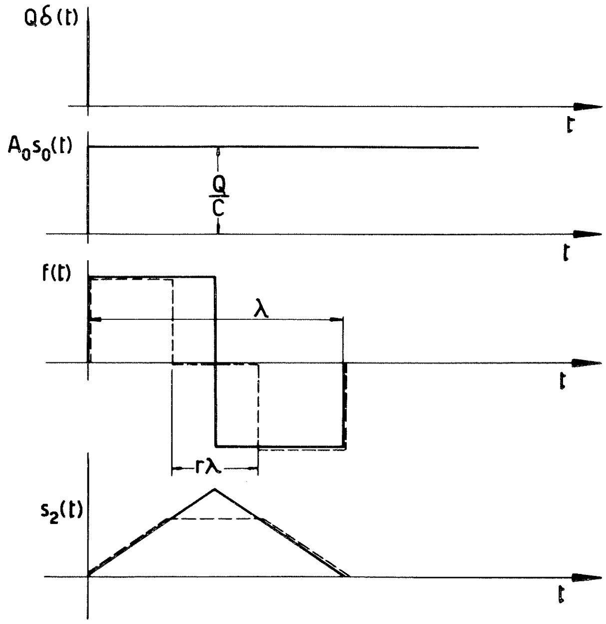

Fig. 2 - Waveforms in the case a): delta function input current - white noise.

The signal to noise ratio equivalent to (1) is given by:

$$\eta^2 = \frac{[\max s_2(t)]^2}{\int_0^\infty N_1 |f(\omega)|^2 \mathrm{d}f} = \qquad (5)$$
$$= \frac{2A_1^2}{N_1} \frac{\left[\int_0^{t_0} f(\tau)\, s_1(t-\tau)\, \mathrm{d}\tau\right]^2}{\int_0^\lambda f^2(\tau)\, \mathrm{d}\tau}$$

where t_0 is the time at which the output signal $s_2(t)$ is maximum.

We must find now $f(t)$ which maximizes η^2 taking into account (3) and (4). $f(t)$ turns out to be (see Appendix):

$$f(t) = G\, s_1(t_0 - t) - e^{t/\sigma} \qquad 0 < t < t_0$$
$$f(t) = -e^{t/\sigma} \qquad t_0 < t < \lambda \qquad (6)$$
$$f(t) \equiv 0 \qquad t > \lambda$$

t_0 is given by (3-A) of Appendix and depends on $s_1(t)$ and on the chosen λ. G is a coefficient given by (2-A) and depends on $s_1(t)$ and λ.

In the $(0, t_0)$ interval, $f(t)$ is obtained, as shown by (6), by the weighted addition of the mirror image of $s_1(t)$ with respect to t_0 and of the mirror image of the exponential decay; for $t_0 < t < \lambda$, $f(t)$ is simply the mirror image of the exponential decay.

TYPICAL RESULTS

a) Let us consider a current pulse $Q(t)$ applied to the input capacity C of an amplifier having white noise spectrum $N_0 = a^2$. Obviously there is no need of network 1; network 2, synthesized according to (6), is characterized by the waveforms of fig. 2; the corresponding η^2 is given by the curve $\lambda/t_1 = \infty$ of fig. 4 (where $A = Q/C$). The resolving time T_2 is equal to λ. This particular result has been already given by Valladas and Levèque [2] according to the theory of Milatz and Keller [3].

The output pulse $s_2(t)$ shows a cuspidal point at the maximum: therefore this amplifier has a large ballistic deficit [1] for an input current pulse of finite width.

If $s(t)$ is chosen as shown in dotted line in the same fig. 2, $s_2(t)$ has a flat top of length $r\lambda$ and as a consequence the ballistic deficit is zero for current pulses whose width does not exceed $r\lambda$. η^2 is reduced by a factor $(1 - r)$ for the same resolving time λ. The network characterized by this new

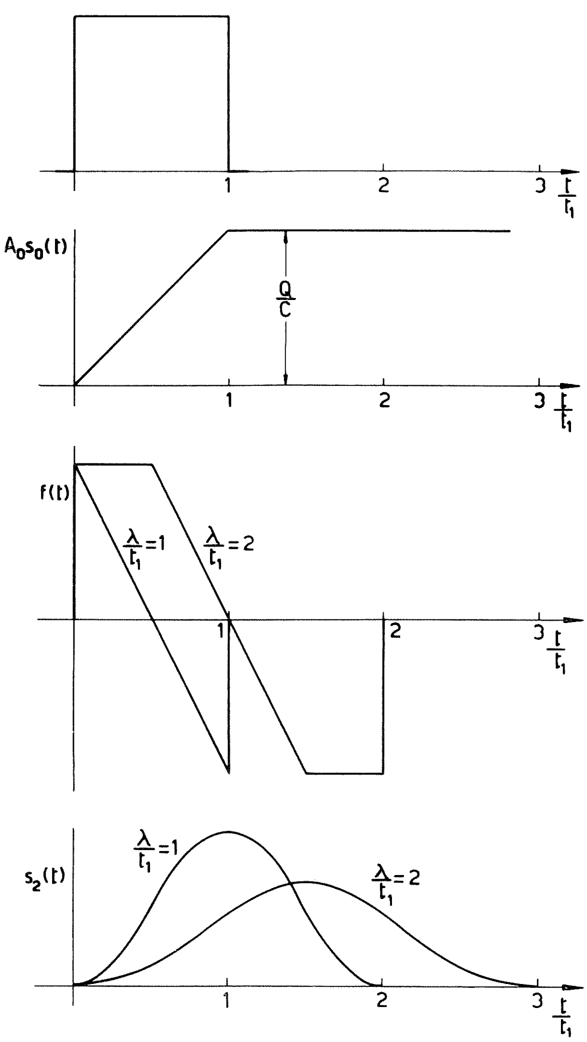

Fig. 3 - Waveforms in the case b): rectangular input current pulse - white noise.

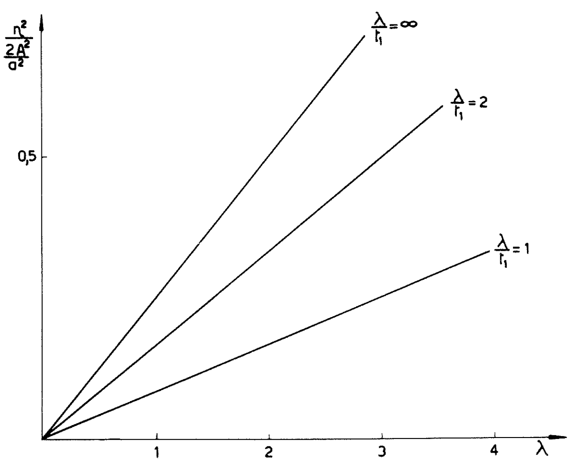

Fig. 4 - Signal-to-noise ratios: vs. λ for the cases a) and b).

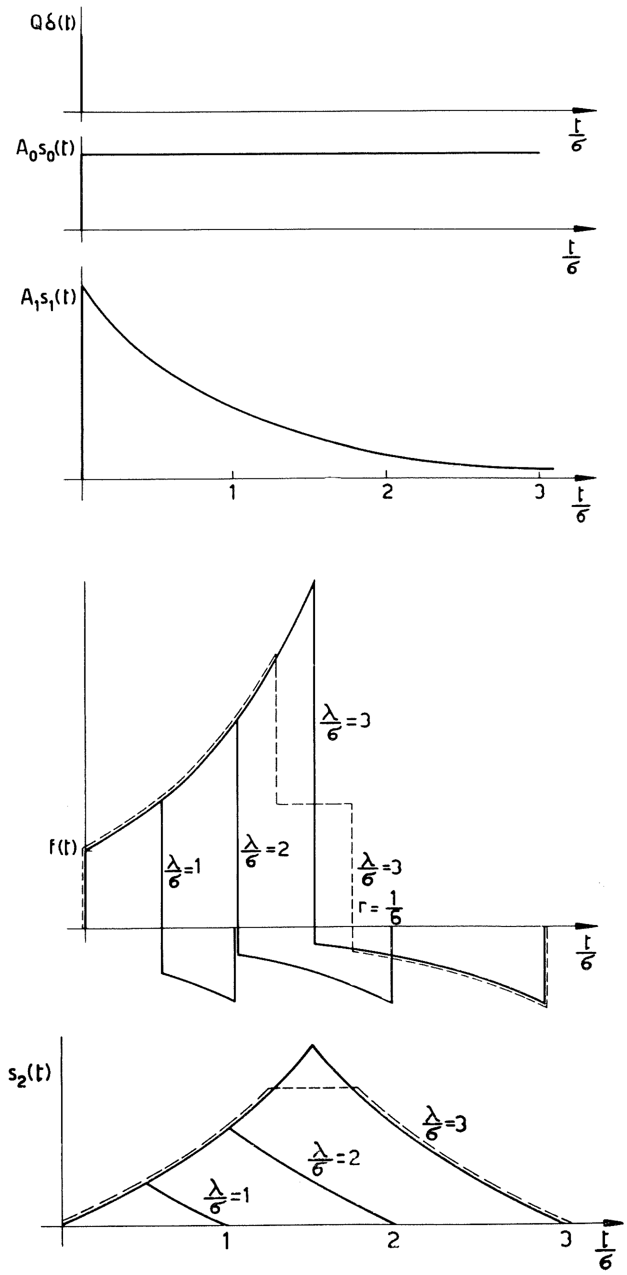

Fig. 5 - Waveforms in the case c): delta function input current - noise spectrum $a^2 + \frac{g^2}{\omega^2}$.

$f(t)$ can be shown to be the « optimum » one when the additional requirement of a flat top is added.

b) Let us consider now a rectangular input current pulse of width t_1 and a white noise N_0.

The application of (6) and (5) leads to the waveforms of fig. 3 and to the $\eta^2(\lambda)$ of fig. 4.

(For $\lambda/t_1 = 1$ we find again the « optimum » network of Valladas and Levèque [2] for a slope measurement on a ramp function).

The resolving time is $T_2 = t_1 + \lambda$. The output waveform has a top with a finite bending radius and the peak amplitude variation, for a change in the input current pulse width, is given by:

$$\frac{\Delta s_{2\,max}}{s_{2\,max}} = -\frac{1}{2}\,\frac{s_2''(t = t_0)}{s_2(t = t_0)}\,(\varrho^2 - \varrho_0^2) \qquad (7)$$

where $s_2(t)$ is the calculated output pulse (fig. 3). ϱ is the mean squared width of the input current pulse, while $\varrho_0 = t_1/\sqrt{12}$ is the corresponding value for the reference current pulse.

The (7) is an easy generalization of an expression given in ref. 1 for the ballistic deficit.

For instance in the case $\lambda/t_1 = 2$ the (7) gives:

$$\frac{\Delta s_{2\ max}}{s_{2\ max}} = -\frac{1}{31}\left(\frac{\varrho^2}{\varrho_0^2} - 1\right)$$

c) We consider thirdly a current pulse $Q(t)$ and a noise reduced to the input given by:

$$N_0(\omega) = a^2 + \frac{g^2}{\omega^2} \qquad (8)$$

The network 1, an *RC* differentiating network with a time constant $\sigma = RC = a/g$, transforms the noise spectrum $N_0(\omega)$ into a white one $N_1 = a^2$; correspondingly the step function $s_0(t) = Q/C.\,1(t)$ (fig. 6) is transformed into $s_1(t) = Q/C\ e^{-t/\sigma}$.

The resulting waveforms calculated from (6) are shown in fig. 5; from (5) η^2 as a function of λ is calculated and plotted in fig. 7 curve 2.

The asymptotic value of $\eta^2\ (\lambda/\sigma \to \infty)$ is equal to η_∞^2 given in ref. 1.

The resolving time T_2 is equal to λ.

Also in this case the output waveform has a cuspidal point. We can reduce the ballistic deficit to zero for a given input pulse width, introducing an identically zero part in the delta response of the cascaded networks 1 and 2.

The delta response of the network 2 becomes the one shown in dotted line in figure 5 for the case $\lambda/\sigma = 3$, $r = 1/6$.

The reduction in η^2 due to the required flat top is easy calculable.

d) As last case we consider an input current pulse $i(t) = Q/\sigma_1 \cdot e^{-t/\sigma}$, and a noise spectrum $N_0(\omega) = a^2 + g^2/\omega^2$.

The waveforms are given in figure 6 and the values of η^2 are plotted in figure 7 for the case $\sigma_1/\sigma = 0.5$.

The resolving time T_2 is practically $T_2 = \lambda + 3\,\sigma_1$.

CONCLUSIONS

The results obtained and mainly the graphs of figures 4 and 7 allow to evaluate the optimum η^2 for many practical cases when a fixed resolving

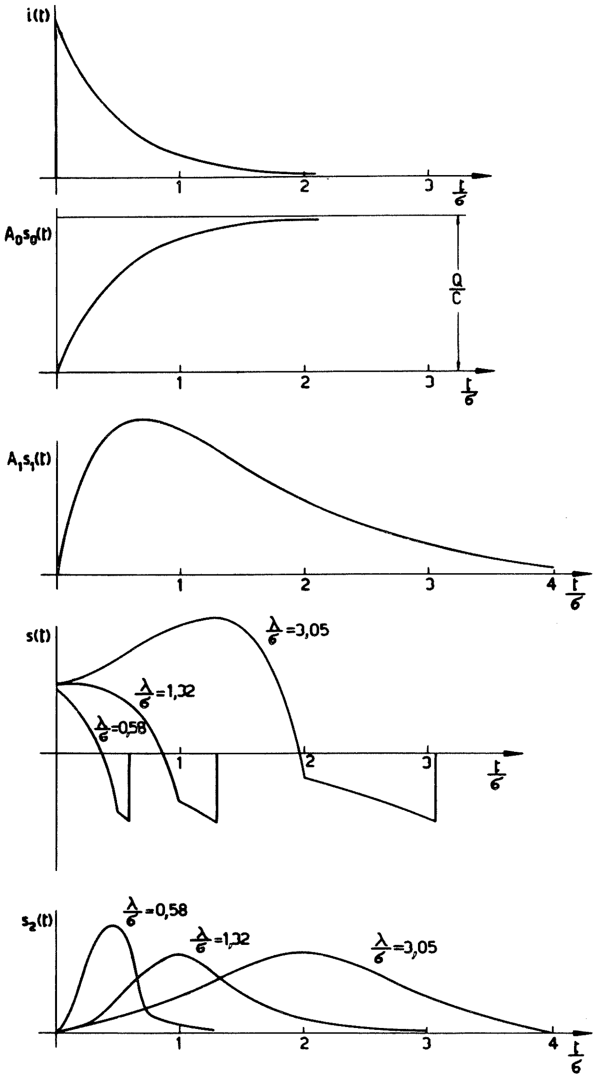

Fig. 6 - Waveforms in the case d): exponential input current pulse $\frac{Q}{\sigma_1}\,e^{-t/\sigma_1}$ noise spectrum $a^2 + \frac{g^2}{\omega^2}$.

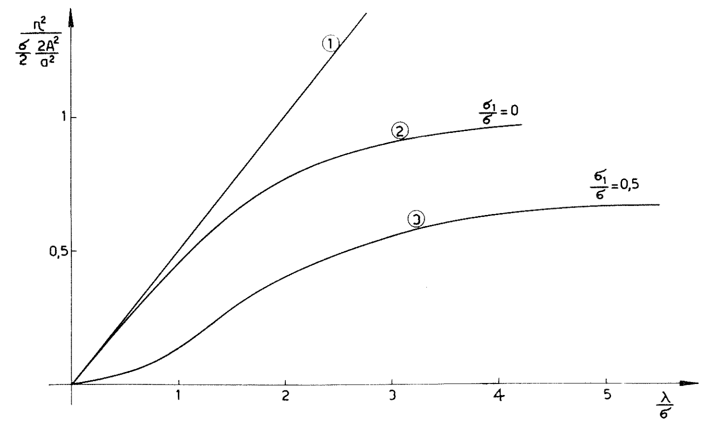

Fig. 7 - Signal-to-noise vs. $\frac{\lambda}{\sigma}$: 1. for the case a); 2. for the case c); 3. for the case d) with $\frac{\sigma_1}{\sigma} = 0.5$.

time is imposed; it is interesting to compare these values, depending on the output width constraint, with the asymptotic ones for $T_2 \to \infty$. The delta responses $f(t)$ of network 2 suggest the synthesis of practical network suitable for the different cases.

The η^2 for these networks are to be compared with the calculated « optima » ones and their ratio is a figure of merit of the considered network.

It is important to remember that the integral condition (4) must be satisfied even by the practical network response in order to cancel at the output the exponential decay of the input.

APPENDIX

$f(t)$ is the function which maximizes the signal to noise ratio (5) taking into account the constraint (4). The t_0 value appearing in (5) is firstly considered as given: and a set of functions $f(t, t_0)$ is obtained

$$f(t, t_0) = G(t_0)\, s_1(t_0 - t) - e^{t/\sigma} \qquad t < t_0$$

$$f(t, t_0) = - e^{t/\sigma} \qquad t_0 < t < \lambda \tag{1-A}$$

where

$$G(t_0) = \frac{\sigma \left(e^{2\lambda/\sigma} - 1\right)}{2 \int_0^{t_0} e^{\tau/\sigma} s_1(t_0 - \tau)\, d\tau} \tag{2-A}$$

The required $f(t)$ is the one of this set which maximizes (5) with respect to t_0. This t_0 is given by

$$s_1^2(t_0) = \frac{2}{\sigma\left(e^{2\lambda/\sigma} - 1\right)} \frac{d}{d\, t_0} \left\{ \int_0^{t_0} e^{\tau/\sigma} s_1(t_0 - \tau)\, d\tau \right\}^2 \tag{3-A}$$

The way in which the (5) is written takes into account the constraint (3). ■

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sommario

RAPPORTO SEGNALE-RUMORE E TEMPO RISOLUTIVO NEGLI AMPLIFICATORI DI IMPULSI PER RIVELATORI NUCLEARI

I limiti superiori del rapporto segnale-rumore sono stati teoricamente calcolati per amplificatori misuratori di carica quali quelli usati in connessione con rivelatori di radiazione tenendo conto del vincolo addizionale di un assegnato tempo risolutivo.