A programmable electronic circuit for modelling CO₂ laser dynamics

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(Received 9 July 2005; accepted 18 September 2005; published online 16 November 2005)

Systems of nonlinear differential equations may be used to model the dynamics of lasers. The study of these systems allows the theoretical investigation of the properties of lasers, their bifurcation analysis, and their control. In particular, the model of the CO₂ laser gives a qualitative explanation of the homoclinic chaos observed in real experiments, characterized by spikes of intensity separated by irregular time intervals. In this paper we use a programmable device to build an electronic implementation of this model and to study the effects of noise in this circuit.

I. INTRODUCTION

Lasers with externally accessible tuning parameters show a variety of dynamical behaviors. In particular, chaos can be observed in lasers with a feedback loop in which cavity losses are modulated by a signal proportional to the output intensity. For instance, in Ref. 2 the experimental apparatus based on an acousto-optic modulator for a loss-modulated Nd:YAG laser is described, while in Ref. 3 an electro-optic modulator is used for a CO₂ laser.

The nonlinear behavior shown by these devices is usually modelled by a set of nonlinear equations. A single mode laser requires three variables, namely, the complex electric field, the population inversion, and the complex polarization. However, whenever the time scale of the polarization is much faster than those of the two variables, the dynamical description reduces to two equations; this is the so-called Class B laser. A third equation, due to an external time-dependent modulation or to a feedback, provides the minimal three-dimensional (3D) phase space necessary for chaos. In the case of a molecular laser as CO₂ three additional variables are required to account for molecular exchanges; the extra equations act as low frequency linear filters; the corresponding phase space is six dimensional (6D).

The qualitative aspects of feedback chaos in a Class B laser are already present in 3D; however a detailed quantitative agreement between model and experiment requires the full 6D model. This model is able to take into account homoclinic trajectories occurring over long times.

In this paper an electronic implementation of the 6D CO₂ laser model is introduced. The electronic implementation is based on a programmable hardware which allows the experimental characterization of the model dynamics with a low cost, reconfigurable and fast experimental setup. This circuit thus constitutes an analogue of the laser device and can be studied to derive conclusions on the laser itself. Moreover, the investigation of the dynamical behaviors of this model has an additional value due to the fact that the dynamical equations considered in the following may be also considered as a model for neuronal spiking with chaotic interspike intervals. The electronic circuit modelling the CO₂ laser dynamics is discussed and experimental results are shown. Furthermore, the effects of noise have been experimentally studied. The presence of noise alters the dynamical time scales of the system, influencing the distribution of interspike intervals. This experimental finding may have important theoretical consequences in the neuronal analogue of this model, which operates in a noisy environment. The plausibility of this type of chaos for coding neuron information and spreading over wide brain domains by neuron synchro-
laser inversion, the rotational manifolds coupled to the resonant levels, and proportional to the difference and sum of the populations of k proportional to the laser intensity, 

\[
\begin{align*}
\dot{x}_1 &= k_0 x_1 (x_2 - 1 - k_1 \sin^2 x_6), \\
\dot{x}_2 &= -\Gamma_1 x_2 - 2k_0 x_1 x_2 + \gamma x_3 + x_4 + P_0, \\
\dot{x}_3 &= -\Gamma_1 x_3 + \gamma x_2 + x_4 + P_0, \\
\dot{x}_4 &= -\Gamma_2 x_4 + \gamma x_5 + z x_2 + z P_0, \\
\dot{x}_5 &= -\Gamma_2 x_5 + \gamma x_4 + z x_3 + z P_0, \\
\dot{x}_6 &= -\beta x_6 + \beta B_0 - B_0 \frac{R x_1}{1 + \alpha x_1},
\end{align*}
\]

where the normalized variables represent the physical variables of the laser, and in particular, \(x_1\) is the photon number proportional to the laser intensity, \(x_3\) is proportional to the laser inversion, \(x_5\) is proportional to the sum of the populations of the laser resonant levels, \(x_4\) and \(x_5\) are, respectively, proportional to the difference and sum of the populations of the rotational manifolds coupled to the resonant levels, and \(x_6\) is a term proportional to the feedback voltage which acts on the cavity loss. The other parameters are connected to the physics of the laser and are chosen as follows: \(k_0 = 28.5714\), \(k_1 = 4.5556\), \(\gamma = 0.05\), \(P_0 = 0.016\), \(\alpha = 32.8767\), \(\beta = 0.4286\), \(\Gamma_1 = 10.0643\), \(\Gamma_2 = 1.0643\).

As shown in the simulation trend of Fig. 1, the laser intensity exhibits spikes separated by irregular return times. Irregular spike intervals are a typical feature of Shilnikov chaos, which is due to the perturbation of an homoclinic orbit associated with a saddle focus fixed point of system in Eq. (1). The return times strongly depend on the initial point in which the trajectory enters the unstable manifold of the saddle focus and thus are irregular.

### III. THE DEVELOPMENT SYSTEM

The adopted device consists of a Field Programmable Analog Array (FPAA), the AN221E04 FPAA, which can be programmed by a software development tool called AnadigmDesigner2.

The core of the device is a two-by-two matrix of blocks named CAB (Configurable Analog Blocks) that can be connected with each other and with external I/O blocks. Each CAB contains a digital comparator, an analog comparator, two operational amplifiers and a set of capacitors. The programmability of FPAA systems is in fact provided by the switched capacitor technology.

The CAB blocks are surrounded by the other subsystems. One section is dedicated to clock management, another to signal I/O and a digital section is devoted to IC configuration and dynamic reprogrammability. The digital section is based on a Configuration Logic State Machine mapping the interconnections inside the IC. This state machine allows the FPAA to be dynamically reprogrammed. A look-up-table provides the possibility of implementing a CAM generating arbitrary wave forms.

The number of CABS is four for each board, thus a single board can provide the possibility of implementing relatively simple dynamics. However, the Anadigm devices can be connected to each other in a very simple way to build more complex circuits. One of these boards is connected to the serial port and allows the other boards to be programmed. The modularity of these boards allows to expand the number of CAMs available for its own design.

The software development tool allows the FPAA to be connected with the I/O ports and the desired circuit to be designed using predefined blocks. There is a large number of
blocks called CAMs (Configurable Analog Modules) performing different functions.

The main blocks useful to implement nonlinear circuits are gain blocks, sum blocks, integrators, filters, multipliers.

The first, and simplest, is the GAININV block that allows us to fix a gain (in the range 0.01–100), and an internal clock for switch driving. Another block is SUMINV. This block integrates its inputs by using a fixed time constant and outputs the resulting signal. Here again it is necessary to fix the clock time for the internal switching. This block provides up to three input weights that can be selected independently for each input. Another CAM allows analog filters to be implemented. The VOLTAGE block provides a voltage reference and can be used to realize bias terms. Finally, there are many other CAMs that are useful in the design of analog circuits.

The internal operations of the FPAA are performed through voltage signals in differential mode. Thus, starting from the dimensionless equations of the dynamical system, an equivalent circuit is designed by assuming that the state variables are voltage signals.

The dynamic range of signals inside the FPAA device is bounded by physical constraints, so that in general the state variables must be scaled.

Therefore, to design a new nonlinear circuit in the FPAA framework, the following steps should be accomplished:

(i) Determine the dynamic range of the variables of the dimensionless equations. Define new scaled variables according to the FPAA bounds.
(ii) Construct a block scheme of the circuit, paying attention to the dynamic range bounds that must be respected in all the blocks of the circuit.
(iii) Design the FPAA schematic by substituting idealized blocks with FPAA blocks.
(iv) Run the FPAA device. Some parameters may need to be adjusted by trial and error.

In Ref. 10 the approach has been successfully applied to the implementation of a Chua’s circuit.

IV. IMPLEMENTATION OF THE MODEL

Equations (1) are dimensionless and thus require some preliminary operations to be implementable on the FPAA device. First of all, to reduce the complexity of the circuit we observed that in the dynamic range of the variables it can be assumed that \( \sin^2 x \approx x^2 \). This allows the use of a nonlinearity which can be implemented by standard FPAA CAMs.

Equations (1) are then rescaled to fit the dynamic range of FPAA internal voltages. The new rescaled variables are defined by the following relationships:

\[
X_1 = S_1x_1, \quad X_2 = x_2, \quad X_3 = x_3, \\
X_4 = S_4x_4, \quad X_5 = S_5x_5, \quad X_6 = x_6,
\]

with \( S_1 = 200, \quad S_4 = \frac{1}{10}, \quad S_5 = \frac{1}{10} \).

This leads to the following equations:

\[
\dot{X}_1 = k_0X_1(X_2 - 1 - k_1X_6), \\
\dot{X}_2 = -\Gamma_1X_2 - G_1\frac{X_4}{S_4} + \gamma X_2 + \frac{X_4}{S_4} + P_0, \\
\dot{X}_3 = -\Gamma_1X_3 + \gamma X_2 + \frac{X_5}{S_5} + P_0, \\
\dot{X}_4 = -\Gamma_2X_4 + \frac{S_4X_4}{S_5} + zS_4X_2 + zS_4P_0, \\
\dot{X}_5 = -\Gamma_2X_5 + \frac{S_5X_4}{S_4} + zS_5X_3 + zS_5P_0, \\
\dot{X}_6 = -\beta X_6 + \beta B_0 - \beta \frac{X_3}{S_1}.
\]

Finally, in order to account for some inaccurate parameter implementations due to the reprogrammable device, we introduce three further parameters in our equations \( G_1, G_2, \) and \( G_3 \), so that the model implemented on the analog device can be written as follows:

\[
\dot{X}_1 = k_0X_1(X_2 - 1 - k_1X_6), \\
\dot{X}_2 = -\Gamma_1X_2 - G_1\frac{X_4}{S_4} + \gamma X_2 + \frac{X_4}{S_4} + P_0, \\
\dot{X}_3 = -\Gamma_1X_3 + \gamma X_2 + \frac{X_5}{S_5} + P_0, \\
\dot{X}_4 = -\Gamma_2X_4 + \frac{S_4X_4}{S_5} + zS_4X_2 + zS_4P_0, \\
\dot{X}_5 = G_2\left(-\Gamma_2X_5 + \gamma \frac{S_5X_4}{S_4} + zS_5X_3 + zS_5P_0\right), \\
\dot{X}_6 = G_3\beta\left(-X_6 + B_0 - \beta \frac{X_3}{S_1}\right).
\]

All the circuit parameters are programmable, but their values cannot be accurately fixed. Moreover, they are implemented with some parameter tolerances. The parameters \( G_i \) are experimentally tuned to compensate these inaccuracies.

The circuit schematic is shown in Fig. 2. Since each Anadigm board has a limited number of available CAMs, three Anadigm boards are required to implement Eqs. (4). The implementation follows the design methodology based on operational amplifier blocks and through the use of CAM standard blocks is quite intuitive. Sum filter blocks, labelled in Fig. 2 as SF, implement the sum of the right-hand terms of each of the six equations (4).

For instance, SF1 block implements the first equations of (4), i.e., that referred to the variable \( x_1 \). For simplicity, let us indicate with \( I_1 \) and \( I_2 \) the nonlinear terms in the \( x_1 \) equation,
i.e., \( I_1 = k_0 x_1 x_2 \) and \( I_2 = -k_0 k_1 x_2^2 \). The \( \dot{x}_1 \) equation in (4) becomes \( \dot{x}_1 + k_0 x_1 = I_1 + I_2 \), which is exactly what the sum filter block implements. In fact, the sum filter block\(^9\) has a transfer function equal to \( G(s) = \frac{I_1(s) + I_2(s)}{X(s)} = \frac{1}{s + k} \). The inputs of the SF1 block are the signal \( x_1 x_2 \), generated by the multiplier block \( M1 \) which in turn has as inputs \( x_1 \) and \( x_2 \), and the signal \( x_2^2 \) which is the output of the multiplier block \( M2 \). The gain factors \( k_0 \) and \(-k_0 k_1\) are realized inside the block SF1. The block H1 is a holder block required to match the switched-capacitor phases of the output of the block M1 with the block SF1. The other equations are implemented in a similar way.

V. EXPERIMENTAL RESULTS

Several experiments have been performed to investigate the behavior of the circuit. The experiments discussed in this section refer to the behavior of the circuit for different parameters and to the effects of noise on the dynamics. All the data have been acquired by using a data acquisition board.
(National Instruments AT-MIO 1620E) with a sampling frequency $f_s=50$ kHz and subsequently plotted by using MATLAB.

The first set of experiments has dealt with the investigation of the behavior of the circuit for different parameter values. The results confirm the theoretical prediction of several windows of chaotic behavior alternated with periodic behavior.[11] Figures 3 and 4 refer to two different sets of parameters leading to chaotic behavior. The wave forms of the signal $x_1$ are shown in Figs. 3(a) and 4(a), while the phase-plane $x_1-x_6$ is shown in Figs. 3(b) and 4(b). Figures 3(c) and 4(c) show the distribution of interspike intervals. The chaotic behavior of the circuit is clearly visible in the distribution of $P(T)$ where the broad range of interspike intervals displayed by the circuit is evident. The main differences between the two data sets rely in the amplitude of the small oscillations which follow the spike. This leads to a different form of the attractor in the phase plane, where it is evident that in the second case (Fig. 4) the chaotic behavior

FIG. 3. Experimental results referring to the behavior of the circuit with parameters $G_1=28.6$, $G_2=36.8$, $G_3=73.3$, $R=164$, and $B_0=0.57$. (a) Trend of the intensity variable $x_1(t)$. (b) Phase plane $x_1-x_6$. (c) $P(T)$ (log scale).

FIG. 4. Experimental results referring to the behavior of the circuit with parameters $G_1=28.6$, $G_2=36.8$, $G_3=7.4$, $R=209.2$, and $B_0=0.63$. (a) Trend of the intensity variable $x_1(t)$. (b) Phase plane $x_1-x_6$. (c) $P(T)$ (log scale).
is concentrated in a smaller region; since the time spent close to the saddle point is reflected in the distribution of interspike intervals, this also leads to a different distribution $P(T)$. The second set of experiments refers to the experimental characterization of the effects of noise in the circuit. The model under investigation is described by Eqs. (4) with the last equation modified as follows:

$$\dot{X}_6 = G_3 \beta - X_6 + \beta B_0 - \beta \frac{X_1}{S_1} + G_4 D \xi(t),$$

where $\xi(t)$ is a Gaussian noise with cutoff frequency of 25 kHz generated by the National Instrument board, which for our purposes can be considered as white Gaussian noise.

A theoretical detailed description of the effects of noise in the laser system is given in Ref. 11. The main effects of the noise are the changes in the time scales of the model. The interspike interval depends on the time the trajectory spends close to the saddle point; when noise is present, the trajectory cannot come closer to the saddle point than the noise level, thus the mean average interspike interval $T_0(D)$ decreases with increasing noise. Moreover, the overall distribution $P(T)$ of the interspike interval $T$ changes in presence of noise. When noise is absent (i.e., when only intrinsic circuit noise is present), $P(T)$ has several peaks [Fig. 5(a)]. When noise is added, $P(T)$ becomes characterized by a dominant peak [Fig. 5(b)]. When noise is further increased, $P(T)$ is characterized by a smaller dominant peak and a greater variance [Fig. 5(c)]. Figure 5 also shows the waveform of the circuit variable $x_1(t)$ and the mean average interspike interval $T_0(D)$ which decreases with increasing $D$.

Finally, some very important experimental results concerning synchronization of two laser circuits by using a common noise source are reported. Recently, it has been pointed out that complete synchronization of two identical chaotic circuits driven by a common noise source can occur if a significant contraction region in the phase space does exist. Moreover, in Ref. 12 phase synchronization of two nonidentical chaotic systems driven by a common noise source has been numerically shown.

In our system the structure of the phase space is characterized by a small expansion region [Fig. 4(b)] around the saddle point. Two nonidentical circuits were thus considered and noise-induced synchronization has been experimentally investigated. The two circuits have different parameters due to intrinsic tolerances of the device. Moreover, two different values of the parameter $R$ have been chosen ($R=258.6$ for the first circuit and $R=237.2$ for the second circuit). The circuits were driven by a common noise signal generated by a function generator (HP 33120A), which provides a Gauss-
ian noise signal with cutoff frequency of 10 MHz. To evaluate phase synchronization between the two circuits, the following phase variable is defined:

\[ \phi(t) = 2\pi \left( k + \frac{t - \tau_k}{\tau_{k+1} - \tau_k} \right), \]

where \( \tau_k \) is the spiking time. The phase difference between the two circuits is then evaluated

\[ \Delta \phi(t) = \phi_1(t) - \phi_2(t). \]

The results are shown in Fig. 6, where \( \Delta \phi(t) \) and wave forms \( x_1(t) \) of the two circuits are reported for three different noise values. As it can be noticed for an optimal noise level, the two circuits show phase synchronization, i.e., \( \Delta \phi(t) = \text{const} \). Phase synchronization is lost for greater or lower values of the noise. Correspondingly, as shown in Fig. 7, which gives the distribution \( P(T) \) of the interspike intervals of the two circuits, a regularization of the interspike interval distribution occurs.

VI. CONCLUSIONS

In this paper a programmable analog circuit implementing CO\(_2\) laser dynamics is introduced. The design and the implementation of the circuit are based on an analog programmable device, the FPAA, which provides the possibility of experimentally exploring the main features of CO\(_2\) laser dynamics in a fast and low cost way.

In particular, the homoclinic chaos typically shown by these lasers when driven in a feedback loop has been experi-
mentally observed and the distribution of interspike intervals has been characterized. Moreover, the effects of noise in these circuits have been investigated from an experimental point of view and very interesting results have been shown. It has been shown that two laser circuits with different parameters and with intrinsic device mismatches may show phase synchronization when driven by a common noise source with a suitable noise level. This has important theoretical consequences. In fact, the dynamics of CO₂ lasers exhibits some typical features of biological neurons characterized by irregular interspike intervals. On the other hand, in Ref. 13 experimental observations of noise-induced regularization of neurocortical neuron dynamics have been reported.

ACKNOWLEDGMENTS

This work was partially supported by the Italian “Minis-tero dell’Istruzione, dell’Università e della Ricerca (MIUR) under the projects Firb RBNE01CW3M and by the EU under the COST ACTION B27 “ENOC.”


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