ELSEVIER

15 March 2001

Optics Communications 189 (2001) 313–319

OPTICS
COMMUNICATIONS

www.elsevier.com/locate/optcom

Negative hysteresis in a laser with modulated parameters

A.N. Pisarchik [a,*,1], B.F. Kuntsevich [b], R. Meucci [c], E. Allaria [c]

[a] [illegible]
[b] [illegible]
[c] [illegible]

Received 26 May 2000; received in revised form 6 October 2000; accepted 15 December 2000

Abstract

The negative hysteresis at low frequencies (less than 10 Hz) is observed in a CO_2 laser with modulated discharge current. The origin of this phenomenon is found in an additional loss modulation which can appear due to the heating of intracavity elements by laser radiation. The results of numerical simulations on the base of the complex laser model are in a good agreement with experimental results. © 2001 Published by Elsevier Science B.V.

[illegible] 05.45.Gg; 05.45.Ac; 42.55.Lt; 42.65.Sf

[illegible] CO_2 laser; Parameter modulation; Hysteresis

1. Introduction

In many experiments with lasers a control parameter is slowly varied in time [1–3]. When the parameter is swept forth and back in the vicinity of a bifurcation point the hysteresis is observed. This phenomenon occurs in any event: when the laser is brought through the first laser threshold (lasing switch on and switch off) [4,5] or when the bifurcation parameter passes through period-doubling or saddle-node bifurcation points [1,2,6]. Coexistence of multiple states usually leads to hysteresis in the field of bistability. The origin of this hysteresis lies in the delay of a bifurcation when the parameter is swept across the region where the bifurcation point is believed to lie [7,8]. When a laser parameter is modulated periodically, both first and second laser thresholds as well as bifurcation points (period doubling and saddle node) are shifted resulting to dynamical tracking of unstable periodic orbits [9,10] or to the deformation of attractor boundaries [11]. The hysteresis depends on the frequency of the parameter change and typical hysteresis is positive, i.e. when the laser is switched on, the lasing occurs at higher value of a pump parameter or at lower value of losses, than when the laser is switched off. However, at certain conditions it does not happen always and researchers should be careful in a treatment of experimental results obtained with parametrically modulated lasers. In particular, a very slow change in a parameter can produce negative hysteresis of lasing state. This phenomenon has been earlier observed by Tredicce et al. [1] in experiments with

---

[*] Corresponding author. Fax: +52-47-175-000.

[illegible] apisarch@foton.cio.mx (A.N. Pisarchik).

[1] On leave from Stepanov Institute of Physics, National Academy of Sciences of Belarus, Minsk, Belarus.

0030-4018/01/$ - see front matter © 2001 Published by Elsevier Science B.V.
PII: S0030-4018(01)01011-2

a loss-modulated CO_2 laser but has not been properly treated.

In this paper we will show that the origin of the negative hysteresis lies in a difference in the temperature of laser intracavity elements for the lasing and non-lasing states that causes a phase delay in the laser response. It should be noted that similar thermal effects have been recently observed by Suret et al. [12] in an optical parameter oscillator. Here we reveal this behavior in a CO_2 laser with periodic modulation of the discharge current. We demonstrate this phenomenon with experiments and numerical simulations and discuss its possible mechanisms.

The paper is organized as follows. In Section 2 we present experimental results obtained in a CO_2 laser with modulated discharge current. The model equations, numerical results, and discussions are given in Section 3. Finally, the conclusions are presented in Section 4.

2. Experiment

The experiments have been carried out with a single mode CO_2 laser. The experimental setup has already been described elsewhere [13,14]. The laser contained an electro-optical CdHgTe modulator and ZnSe Brewster windows on the active tube. The hysteresis phenomena have been studied by applying the periodic triangular modulation to the discharge current in the laser with a feedback signal to the modulator from a power detector.

In the left-hand column of Fig. 1 we show the signal of the current (upper curves) and the laser response (lower curves) for three different modulation frequencies: 1, 200, and 500 Hz. Since we consider hysteresis in a lasing state, we normalize hysteresis to the duration of the lasing state (pulse duration), i.e. we use the following definition $H = (h_1 - h_2)/\tau$, where h_1 and h_2 are the time intervals from the moment of the minimal current to the lasing onset and offset, respectively, as indicated in Fig. 1(a), and τ is the duration of the laser pulse. Thus, the modulation period $T = h_1 + h_2 + \tau$. The normalization of the hysteresis to τ allows us to compare quantitatively the hysteresis values for different modulation frequencies ν and laser powers. In the right-hand column of Fig. 1 we plot the hysteresis loop, i.e. the laser intensity versus the pump current. One can see that at the low frequency ($\nu = 1$ Hz) (Fig. 1(a)) the hysteresis is negative, i.e. $h_1 < h_2$, while at higher frequencies it is positive.

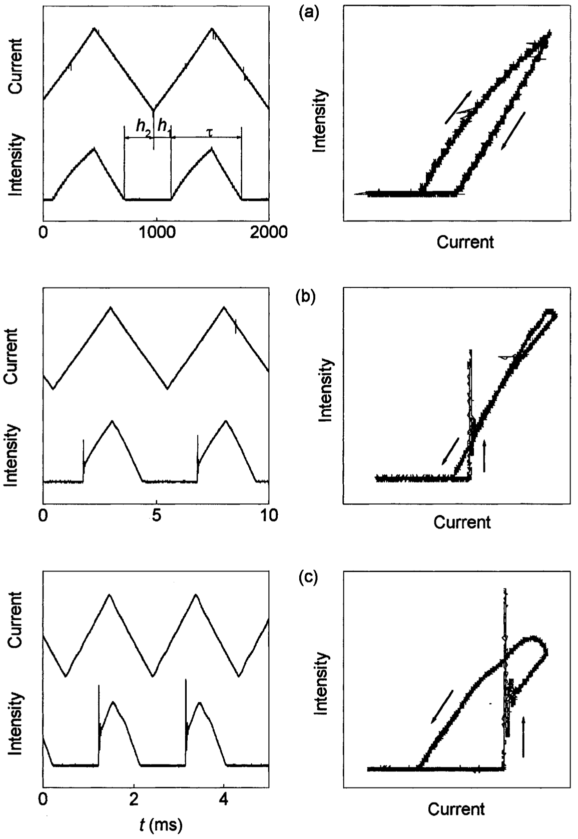

Fig. 1. Experimental discharge current and laser intensity versus time at the modulation frequencies (a) 1, (b) 200, and (c) 500 Hz. The hysteresis loops are shown in the right-hand column.

The frequency dependence of □ is shown in Fig. 2(a). As seen from this graphics, the hysteresis is negative ($H < 0$) at $\nu < 10$ Hz. In Fig. 2(b) we plot the same dependence but in the semilog scale. As seen from this figure, the hysteresis dependence can be approximated by two lines: the straight line for the relatively high frequencies and saturation line for the low frequencies. At lower frequencies □ does not depend on ν. The existence of the negative hysteresis while $\nu \to 0$ can be explained by the temperature difference in the intracavity

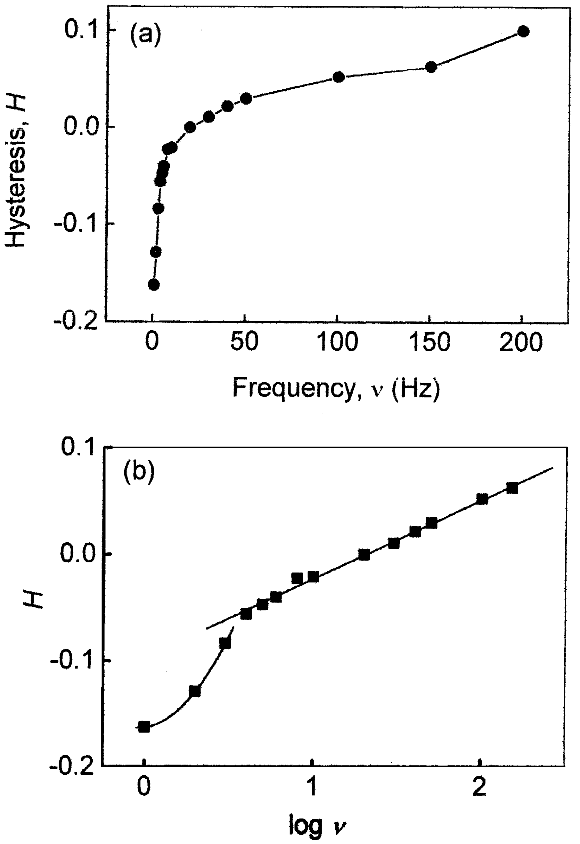

Fig. 2. Experimental frequency dependence of hysteresis H in a CO_2 laser with pump modulation in (a) linear scale and (b) semilog scale. Negative hysteresis observed at $\nu < 10$ Hz is saturated at very low frequencies.

elements when the laser is working and when it is switched off. The laser power leads to the increasing temperature in the semiconductor intracavity elements (ZnSe windows and CdHgTe modulator) that results in their lower transmittance and hence in the increasing cavity losses. The cavity losses are different at the beginning and at the end of the lasing pulse; at the beginning the losses is lower and therefore $h_1 < h_2$. At very low modulation frequencies the temperature equilibrium is established, the temperature difference becomes a constant and does not depend on the frequency. This causes the saturation in H and results in the low limit in hysteresis at which the forward and the backward sweeps give the same results.

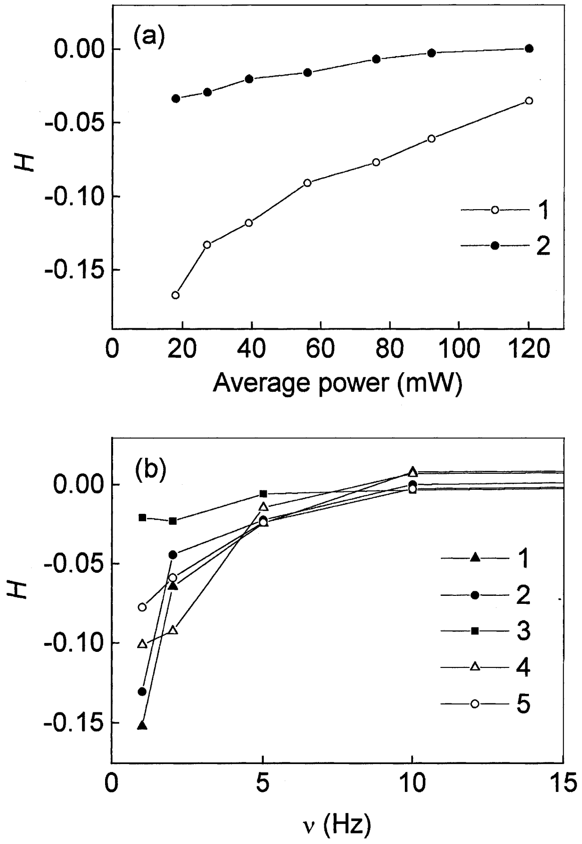

Fig. 3. (a) Experimental dependences of hysteresis on average power (at 100 Hz) for modulation frequencies (1) 1 and (2) 10 Hz. (b) Hysteresis versus modulation frequency at different laser powers (1) 17, (2) 32, and (3) 58 mW and with additional intracavity ZnSe plate at (4) 15 and (5) 30 mW. Hysteresis does not depend on the laser power in the positive part.

While the positive hysteresis depends only on the modulation frequency, the negative part depends also on the laser power. In Fig. 3(a) we show the power dependences for two different frequencies: 1 and 10 Hz. The laser power was regulated by the discharge current. The increasing power leads to a decrease in the absolute value of H. The change in H is provided mainly by the variation of the pulse duration τ, while h_1 and h_2 change slightly. The frequency dependences for different average powers are shown in Fig. 3(b). One can see that the negative part depends strongly on the power. Such a behavior also can be a confirmation of the influence of temperature processes in the intracavity semiconductors elements (electro-optical modulator and Brewster window). In order to check this assumption, we introduced an additional ZnSe plate into the cavity. The results shown by curves (4) and (5) display the different values for the negative part of □. This again confirms our suggestion concerning the thermal mechanism of negative hysteresis.

3. Numerical simulations

[illegible]

For description of a single-mode CO_2 laser we use the following system of equations [6,15,16]:

$$\frac{dN_1}{dt} = \beta_1 N_e N_0 f(t) - W_{10}N_1 + W_{21}N_2 + Bu(n_2^j - n_1^j), \tag{1}$$

$$\frac{dN_2}{dt} = \beta_2 N_e N_0 f(t) + W_{NC}N_{N_2}(N_0M_1 - N_2M_0) - W_{21}N_2 - Bu(n_2^j - n_1^j), \tag{2}$$

$$\frac{dM_1}{dt} = \beta_3 N_e M_0 f(t) + W_{NC}N_{CO_2}(N_2M_0 - N_0M_1), \tag{3}$$

$$\frac{du}{dt} = v\mu[\chi y^j - k_0 - k(t)]u. \tag{4}$$

Here N_0, N_1 and N_2 are the relative quasi-equilibrium populations of the vibrational 00^00, 10^00 and 00^01 levels of CO_2; M_0 and M_1 are the relative populations of the fundamental ($v = 0$) and first exited ($v = 1$) vibrational levels of N_2; W_{NC} is the rate constant of the vibrational energy transfer from N_2 to CO_2; N_{CO_2}, N_{N_2} are the volume densities of CO_2 and N_2; $n_1^j = N_1F_1^j$ and $n_2^j = N_2F_2^j$ are the relative populations of lower and upper laser rotational sublevels of CO_2 (for simplicity the rotational quantum number □ is considered to be the same for both levels); N_e is the free-electron density in the active medium; W_{21} and W_{10} are the effective rates of collisional relaxation in 00^01–10^00 and 10^00–00^00 channels; β_1, β_2, and β_3 are the pumping rates of two vibrational levels of CO_2 and excited level of N_2 in the electric discharge; F_1^j and F_2^j are the normalized Boltzmann functions determining the part of molecules in the corresponding rotational sublevels in thermodynamic equilibrium; □ and χ are the Einstein coefficient and specific gain coefficient at the lasing frequency; □ is the speed of light in the active medium; μ is the packing coefficient for the active medium in the cavity; □ is the average radiation density in the cavity; $y^j = n_2^j - n_1^j$ is the population inversion of rotational sublevels, k_0 and $k(t)$ are the constant and variable cavity losses. The function $f(t)$ describes the triangle shape of the pump modulation (as in the experiments) with the amplitude from 0 to 1.

When a laser operates in a pump-periodic regime, the pump modulation can be accompanied by the additional modulation of the cavity losses. This may be caused by various reasons, for example, by a heating of intracavity elements due to the laser radiation and a change in their transmittance coefficients, by a change in the refractive index of the active medium resulted from varying concentration of the free electrons in the electric discharge, and so on. The variation of the losses, $k(t) = k_1 f(t - \theta)$, is described by the same function as the pump modulation, but with the time delay θ; k_1 is the amplitude of the alternative component of the losses. Thus, θ characterizes the phase shift between the alternative components of the pump and losses.

We also take into account the dissociation of the CO_2 molecules by the following scheme $2CO_2 \leftrightarrow 2CO + O_2$. The fraction of the non-dissociated CO_2 molecules y_c ($y_c = 1$ without dissociation) is described by the following equation

$$\frac{dy_c}{dt} = -k_d N_e N_{CO_2} f(t) + k_r N_{CO} N_{O_2}, \tag{5}$$

where N_{CO_2}, N_{CO} and N_{O_2} are the volume densities of the CO_2, CO and O_2 molecules, k_d and k_r are the constants of the dissociation and recombination of the CO_2 molecule. The average temperature is calculated by the equation [17]

$$\left[\frac{5}{2}\left(1 + \frac{N_{N_2}}{N_{CO_2}} + \frac{N_{CO}}{N_{CO_2}} + \frac{N_{O_2}}{N_{CO_2}}\right) + \frac{3}{2}\frac{N_{He}}{N_{CO_2}}\right]\frac{dT}{dt} = \frac{hv_3 - 3hv_2}{k_B}W_{21}N_2 + \frac{hv_2}{k_B}W_{10}N_1 + \frac{x_T JE}{k_B N_{CO}}f(t) - \frac{18.9}{\pi R^2 k_B N_{CO}}\lambda_m(T - T_w), \tag{6}$$

where λ_m is the thermal conductivity of the active gas mixture, T_w is the wall temperature of the discharge tube, k_B is the Boltzmann coefficient, □ is the discharge current density, □ is the electric intensity in the active medium, x_T is the part of the discharge energy spending for heating of the active medium, ν_2 and ν_3 are the frequencies of the banding and asymmetric modes of the CO_2 molecule.

All calculations are performed for the following parameters of the laser system. The active medium CO_2:N_2:He = 1:1:8 (before the discharge switches on) has the pressure of 15 Torr. The laser operates on a single longitudinal mode of the 10P20 line. The following parameters are used in the simulations [18] $\beta_1 = 2 \times 10^{-10}$ $s^{-1}cm^3$, $\beta_2 = 2 \times 10^{-9}$ $s^{-1}cm^3$, $\beta_3 = 10^{-8}$ $s^{-1}cm^3$, $N_e = 2 \times 10^{10}$ cm^{-3}, $k_0 = 5 \times 10^{-3}$ cm^{-1}, $k_1 = 2 \times 10^{-3}$ cm^{-1}. The other parameters are the same as in the experiment.

□□□□ □□□□□□□ □□□ □□□□□□□□□□

In Fig. 4 we present the results of numerical simulations for the same modulation frequencies as in the experiments: 1, 200, and 500 Hz. The dashed line in Fig. 4(a) displays the modulation function of the cavity losses $f(t-\theta)$. We have found that the introduction of the time delay $\theta = 0.25T$ simulates the experimental situation very well and as seen from Fig. 4(a) hysteresis is negative at $\nu = 1$ Hz (compare with Fig. 1(a)). Similar to the experiments we have observed the transition from negative to positive hysteresis when ν is increased (Fig. 4(b) and (c)).

A distinguishing feature of the laser response at the range of negative hysteresis (at $\nu < 10$ Hz) is that the leading edge of the laser pulse is convex up. This feature is showed up in both experimental and numerical curves (see Figs. 1(a) and 4(a)). The reason of such a phenomenon can be revealed from the analysis of the time behavior of the variables shown in Fig. 5. One can see that at $\nu = 1$ Hz the quasi-stationary regime is realized when the gain factor $\chi y^j = k(t)$ (Fig. 5(e)). The increasing pump leads to an increase in the losses with the time delay θ that results in the convex up shape in the laser power. In our calculation we also take into account the variation in the gas temperature (Fig. 5(b)) and in the dissociation of the CO_2 molecules (Fig. 5(c)), that makes alternations in the gain factor, relaxation rates, and inversion population (Fig. 5(d)). It should be noted that any variation in k_d and k_r in Eq. (5) as well as in λ_m and x_T in Eq. (6) did not allow us to obtain the convex up shape of the laser pulse, if we did not take into account the loss modulation $k(t)$ in Eq. (4).

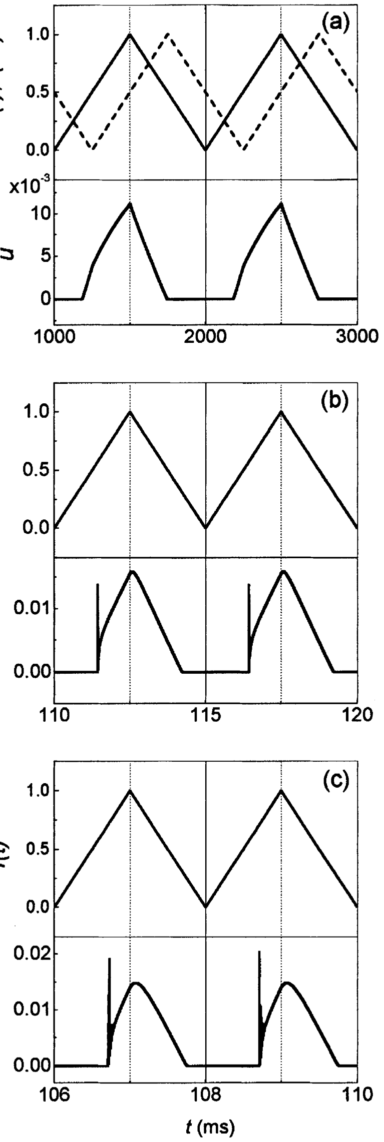

Fig. 4. Numerical time dependences of pump current and laser power at modulation frequencies (a) 1, (b) 200, and (c) 500 Hz. The modulation function of the cavity losses $f(t-\theta)$ is shown by the dashed line.

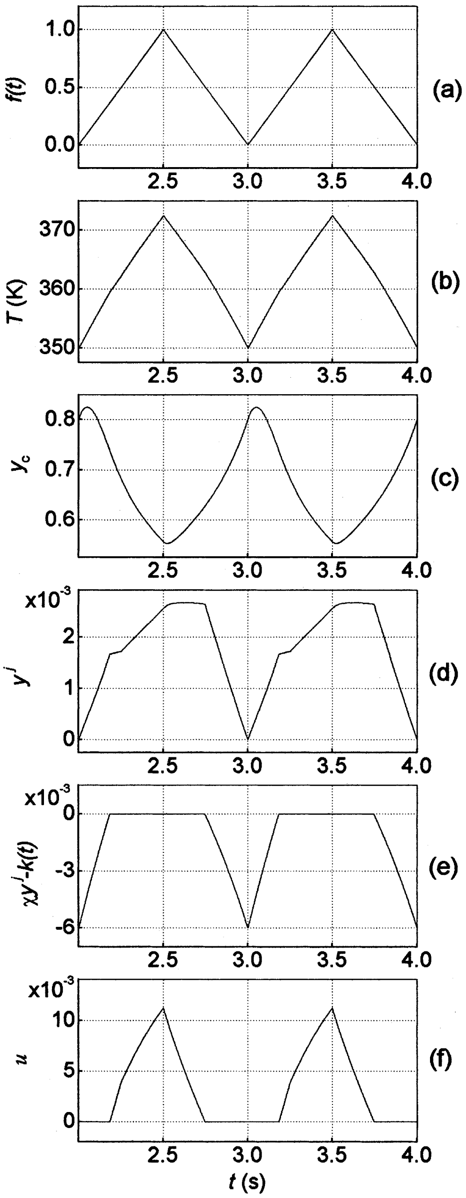

Fig. 5. Numerical time dependences of (a) modulation current, (b) gas temperature, (c) number of non-dissociated CO_2 molecules, (d) inversion population, (e) gain excess over losses, and (f) laser power at $\nu = 1$ Hz.

Now we shall consider the mechanisms by which the pump modulation at low frequencies may produce the additional loss modulation. The possible sources may be either (i) the active medium or (ii) the intracavity elements (modulator, Brewster windows). Let consider each of these mechanisms in more details.

(i) The pump modulation results in the modulation of the free electron density at the active medium and, as a consequence, the modulation of the refractive index. This causes the variation in positions of the longitudinal mode of the resonator and the alternation of the laser frequency and the gain factor. This mechanism would lead to an increase or a decrease of loss depending on the cavity detuning. The sign of such an effect would be reversed according to whether the laser cavity is tuned below or above the maximum gain frequency. Moreover, the order of magnitudes does not fit. Numerical simulations with different parameters of the active medium and discharge current allowed us to exclude this mechanism.

(ii) The transmittance of semiconductor elements depends strongly on the temperature. These elements are heated by the laser radiation and their temperature depends on their dimensions and thermal conductivities. Additional losses may appear due to an increase in the absorption or (and) a disalignment of the intracavity optical elements because of their heating. As seen from the experiments (Fig. 3(b)), the introduction of the additional ZnSe window into the cavity changes the hysteresis in its negative part. Thus, the thermal processes in the intracavity elements do are responsible for the time delay θ.

4. Conclusions

In this work we have studied experimentally and numerically the hysteresis phenomena in a CO_2 laser with modulated discharge current when the laser is switched on and off. We have shown that dependent on the modulation frequency the hysteresis can be either positive or negative. The positive hysteresis is observed at high modulation frequencies (higher than 10 Hz), while the negative hysteresis takes place only at lower frequencies. Here one should distinguish two different mechanisms responsible for appearance of the positive and negative hysteresis. They are respectively a delay of bifurcations and thermal effects. There two mechanisms operate in different time scales;

the delay mechanism dominates at high frequencies while the thermal processes have a determining effect at relatively low frequencies. The results disclose that a possible source of negative hysteresis arises from an additional modulation of cavity losses due to the heating of intracavity elements by the laser radiation. A qualitatively good agreement between experimental and numerical results has been obtained.

Acknowledgements

This work has been supported in part by the grant of the Institute Mexico–USA of the University of California (UC MEXUS) and Consejo Nacional de Ciencia y Tecnología de México (CONACYT). ANP acknowledges support from CONACYT through Cátedra Patrimonial (Ref. 990457). We thank L. Calero for her assistance in the experimental data collection.

References

[1] J.R. Tredicce, F.T. Arecchi, G.P. Puccioni, A. Poggi, W. Gadomski, Dynamic behavior and onset of low-dimensional chaos in a modulated homogeneously broadened single mode laser: experiment and theory, Phys. Rev. A 34 (1986) 2073–2081.

[2] D. Dangoisse, P. Glorieux, D. Hennequin, Chaos in a CO_2 laser with modulated parameters: experiments and numerical simulations, Phys. Rev. A 36 (1987) 4775–4791.

[3] J.C. Celet, D. Dangoisse, P. Glorieux, G. Lythe, T. Erneux, Slowly passing through resonance strongly depends on noise, Phys. Rev. Lett. 81 (1998) 975–978.

[4] F.T. Arecchi, R. Meucci, M. Ciofini, Delayed bifurcations in a CO_2 laser with a slowly swept pump, in: J.H. Eberly, et al. (Eds.), Coherence and Quantum Optics VI, Plenum Press, New York, 1990.

[5] H. Grassi, C. Green, L.M. Hoffer, G.L. Lippi, J.R. Tredicce, Experimental study of the turn-on statistics of class-B lasers under the influence of additive noise, Phys. Rev. A 50 (1994) 805–812.

[6] A.N. Pisarchik, B.F. Kuntsevich, R. Corbalán, Stabilizing unstable orbits by slow modulation of a control parameter in a dissipative dynamic system, Phys. Rev. E 57 (1998) 4046–4053.

[7] P. Mandel, T. Erneux, Laser Lorenz equations with a time-dependent parameter, Phys. Rev. Lett. 53 (1984) 1818–1821.

[8] R. Kapral, P. Mandel, Bifurcation structure of the nonautonomous quadratic map, Phys. Rev. A 32 (1985) 1076–1081.

[9] A.N. Pisarchick, V.N. Chizhevsky, R. Corbalán, R. Vilaseca, Experimental control of nonlinear dynamics by slow parametric modulation, Phys. Rev. E 55 (1997) 2455–2461.

[10] A.N. Pisarchik, Dynamical tracking of unstable periodic orbits, Phys. Lett. A 242 (1998) 152–162.

[11] A.N. Pisarchik, R.Corbalán, Shift of attractor boundaries in a system with a slow harmonic parameter perturbation, Physica D 150 (2001) 14–24.

[12] P. Suret, D. Derozier, M. Lefranc, J. Zemmouri, S. Bielawski, Self-pulsing instabilities in an optical parametric oscillator: Experimental observation and modeling of the mechanism, Phys. Rev. A 61 (2000) 021805-1–021805-4.

[13] M. Ciofini, A. Labate, R. Meucci, M. Galanti, Stabilization of unstable fixed points in the dynamics of a laser with feedback, Phys. Rev. E 60 (1999) 398.

[14] A.N. Pisarchik, R. Meucci, F.T. Arecchi, Discrete homoclinic orbits in a laser with feedback, Phys. Rev. E 62 (2000) 8823–8825.

[15] A.N. Pisarchik, R. Corbalán, V.N. Chizhevsky, R. Vilaseca, B.F. Kuntsevich, Dynamic stabilization of unstable periodic orbits in a CO_2 laser by slow modulation of cavity detuning, Int. J. Bif. Chaos 8 (1998) 1783–1789.

[16] B.F. Kuntsevich, A.N. Pisarchik, Frequency sweeping in a CO_2 laser with modulated losses, J. Opt. B: Quantum semiclass. Opt. 1 (1999) 349–356.

[17] B.F. Gordiets, A.I. Osipov, L.A. Shelepin, Kinetic Processes in Gases and Molecular Lasers, Nauka, Moscow, 1980.

[18] K. Smith, R.M. Tomson, Computer Modeling of Gas Lasers, Plenum press, New York, 1978.